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D. O. MOKHNACH

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Abstract

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D. O. MOKHNACH

THE SIMPLEST MODEL OF HALOS

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Halos are luminous rings in the head of a comet, the center of which practically coincides with the "nucleus" (the photometric center). The radius of a halo grows in proportion to time. The surface brightness of the ring is a decreasing function of time; because of this, halos exist as objects accessible to observation for a comparatively short time—only several days.

Let us suppose that a halo is formed as a result of an isotropic ejection of particles from a point, which we shall call the center of emission. We denote the duration of the ejection by ΔT. We shall regard the emission coefficient n (the number of particles ejected in 1 sec. per unit solid angle) as constant during ΔT. The acceleration of the repulsive force of the Sun, g, may also be regarded as constant, since over the short time of existence of a halo its heliocentric distance changes little, and the dimensions of the halo are small in comparison with the distance to the Sun. The initial velocity of the particles is v_0.

If the origin of a rectangular coordinate system is placed at the center of emission, directing the axis Ox toward the Sun and taking the axis Oz parallel to the line of sight, then, as shown in (1), for a center of emission at rest relative to the Sun, particles ejected simultaneously, after a time t, will be located on a sphere whose equation is

$$(x + {}^1/_2 g t^2)^2 + y^2 + z^2 = v_0^2 t^2 \tag{1}$$

with center at the point $(-{}^1/_2 g t^2, 0, 0)$ and radius $v_0 t$. If the time elapsed from the end of the particle ejection to the moment of observation is T, then the halo is a layer bounded by two spheres with centers at the points

$$(-{}^1/_2 g T^2,\ 0,\ 0), \qquad (-{}^1/_2 g (T + \Delta T)^2,\ 0,\ 0) \tag{2}$$

and radii

$$R_1 = v_0 T, \qquad R_2 = v_0 (T + \Delta T). \tag{2'}$$

In the head of a comet several halos of different diameters are observed simultaneously. The times of their formation differ from one another by only a few hours. Therefore it is expedient to adopt the condition

$$\Delta T \ll T. \tag{a}$$

The small magnitude of the displacement of the halo center relative to the emission center may, owing to condition (a), be taken as equal to

$$\Delta c = -{}^1/{}_2 g T^2. \tag{3}$$

Observations of halos often do not show the displacement of the center that follows from equation (1). This has led some investigators (2) to the conclusion that in halos the molecules are not repelled by the Sun, whereas in the head of the comet the same molecules experience the pressure of solar radiation. This is precisely the so-called paradox of halos.

1. Distribution of the apparent density in the halo

As shown on p. 278 of paper ([1]), the apparent density in the halo is conveniently calculated by the formula

$$N(x,y) = \frac{2n}{g}\int_T^{T+\Delta T} \frac{dt}{t\sqrt{-\frac{1}{4}g^2t^4 + (v_0^2 - gx)t^2 - (x^2+y^2)}} \tag{4}$$

or

$$N(x,y) = \frac{2n}{g}\int_T^{T+\Delta T} \frac{dt}{t\sqrt{v_0^2t^2 - \left(x + \frac{1}{2}gt^2\right)^2 - y^2}}. \tag{4'}$$

Between the lines of intersection of the spheres bounding the halo with the plane xOy, the lower limit of the integrals (4) and (4′) is t_1—the smaller of the two real roots of the radicand trinomial. In this case the integral becomes improper, but exists.

Calculating the integral (4), we find

$$N(x,y) = \frac{4n}{v_0 g t_1 t_2}\left[\operatorname{arc\,tg}\sqrt{\frac{t_1^2(t_2^2 - T^2)}{t_2^2(T^2 - t_1^2)}} - \operatorname{arc\,tg}\sqrt{\frac{t_1^2\left[t_2^2 - (T+\Delta T)^2\right]}{t_2^2\left[(T+\Delta T)^2 - t_1^2\right]}}\right]; \tag{5}$$

t_1 and t_2—the roots of the radicand trinomial in integral (4)—determine the least and greatest times required for a particle to reach the point with coordinates

$(x, y, 0)$ (see ([1]), pp. 274-278). Formula (5) gives the exact expression for the apparent density at all points of the plane xOy onto which the particles forming the halo are projected.

It can be shown that the apparent density attains its greatest value on the circle

$$\left(x + \frac{1}{2}gT^2\right)^2 + y^2 = v_0^2 T^2, \tag{6}$$

i.e., at the intersection of the inner boundary of the halo with the plane xOy.

Let us consider the distribution of the apparent density, for fixed values of T and ΔT, along the x-axis, i.e., putting $y = 0$ in formula (4′). Obviously, as x increases from $x = \frac{1}{2}gT^2$ at the center of the halo to $x = v_0 T - \frac{1}{2}gT^2$ at point A, the integral (4′) increases, since for constant limits T and $T + \Delta T$ the integrand increases. It attains its greatest value in this interval at point A, where $x = v_0 T - \frac{1}{2}gT^2$ ($t_1 = T$), and, by formula (5),

$$N(A) = \frac{4n}{v_0 g t_1 t_2}\left[\frac{\pi}{2} - \operatorname{arc\,tg}\sqrt{\frac{t_1^2\left[t_2^2 - (t_1 + \Delta T)^2\right]}{t_2^2\left[(t_1 + \Delta T)^2 - t_1^2\right]}}\right] \tag{7}$$

or, neglecting ΔT^2,

$$N(A) = \frac{4n}{v_0 g t_1 t_2} \operatorname{arc\,tg}\sqrt{\frac{2t_2^2 \Delta T}{t_1\left[t_2^2 - (t_1 + \Delta T)^2\right]}}. \tag{8}$$

On the basis of condition (a), neglecting in the denominator the quantities $2t_1\Delta T$ and ΔT^2, which are small compared with t_1^2, we obtain the approximate formula

$$N(A) = \frac{4n}{v_0 g t_1 \sqrt{t_1}}\sqrt{\frac{2\Delta T}{t_2^2 - t_1^2}}. \tag{9}$$

At point A of the Ox axis (see ([1]), p. 274), the density (the number of particles per unit volume) is determined by the formula $\rho = n/v_0^2 T^2(v_0 - gT)$. The density decreases in the direction AB, if

$$T < \frac{2}{3}\frac{v_0}{g} \tag{b}$$

(v_0/g is the time required for the particles to reach the vertex of the enveloping paraboloid).

Fig. 1

Figure 1: Fig. 1

Consequently, when condition (b) is satisfied, in the direction AB both the density and the chord along which the particles are counted decrease (Fig. 1). Therefore the maximum visible density in the direction $O'B$ (O' is the center of the inner sphere bounding the halo) is located at point A, where $x = v_0 T - \frac{1}{2} g T^2$. At point B (and on the entire outer boundary of the halo) $N = 0$. As is known (see (1), p. 275),

$$t_1^2 = \frac{2}{g^2}\left(v_0^2 - gx - \sqrt{v_0^4 - 2v_0^2 gx}\right), \quad t_2^2 = \frac{2}{g^2}\left(v_0^2 - gx + \sqrt{v_0^4 - 2v_0^2 gx}\right),$$

therefore, for $x = v_0 T - \frac{1}{2} g T^2$,

$$t_2^2 - t_1^2 = \frac{4v_0^2}{g^2}\left(1 - T \,/\, \frac{v_0}{g}\right).$$

Introducing the dimensionless quantities $\lambda = T \,/\, \frac{v_0}{g}$, $\lambda' = \Delta T \,/\, \frac{v_0}{g}$, we find for the maximum of the visible density at point A

$$N(A) = \frac{2ng\sqrt{2\lambda'}}{v_0^3 \lambda \sqrt{\lambda(1-\lambda)}}. \tag{10}$$

Fig. 1

In a similar way one can verify that, in other directions from the center O' to the edge of the halo, the greatest value of the visible density lies on a circle with radius

$$R = v_0 T \tag{11}$$

and center at the point $\left(-\frac{1}{2} g T^2, 0\right)$. It should be noted that the visible density on this circle is not a constant quantity and, for example, at point D

$$N(D) = \frac{2ng\sqrt{2\lambda'}}{v_0^3 \lambda \sqrt{\lambda(1+\lambda)}}, \tag{10'}$$

since at this point $\rho = n / v_0^2 T^2 (v_0 + gT)$. The ratio of the displacement of the halo center to the radius is

Fig. 2

Figure 2: Fig. 2

$$\frac{|\Delta c|}{R} = \frac{1}{2}T \,/\, \frac{v_0}{g} = \frac{1}{2}\lambda. \tag{12}$$

Since in the direction of the halo center the density at the inner boundary is $\rho = n/v_0^3 T^2$, and the length of the chord is $v_0 \Delta T$, the visible density at the center of the halo is

$$N_c = \frac{2ng\lambda'}{v_0^3 \lambda^2}. \tag{13}$$

Fig. 2

All these conclusions are valid when conditions (a) and (b) are satisfied, i.e., when $\lambda' \ll \lambda$ and $\lambda < 2/3$.

2. Change of the halo brightness with time. Halos are observed over a short time interval during which the heliocentric distance of the comet practically does not change; therefore the change in the brightness of the halo may be attributed entirely only to the factors $N' = 1/\lambda\sqrt{\lambda(1-\lambda)}$ or $N'' = 1/\lambda\sqrt{\lambda(1+\lambda)}$ in formulas (10) and (10′). This change is illustrated by Table 1 and Fig. 2.

Table 1

λ	0.1	0.2	0.3	0.4	0.5
N'	33	12.5	7.3	5.1	4.0
N''	30	10.2	5.4	3.3	2.3

We see that, in order to determine λ from observations, it is necessary to carry out an accurate photometric study of the change in brightness at the point of maximum in the halo with time.

Determination of the quantity λ', characterizing the duration of the outburst, is theoretically possible by comparing $N(A)$ and N_c according to formulas (10) and (13)

$$\lambda' = \left(\frac{N_c}{N(A)}\right)^2 \frac{2\lambda}{1-\lambda}. \tag{14}$$

Use of formula (14) is made difficult by the fact that halos are observed against the background of the comet’ s head, and therefore the small quantity N_c is very difficult to measure unless the bright glow of the comet' s "nucleus" is somehow removed.

The rapid decrease of $N(A)$ and $N(D)$ with increasing λ shows that, apparently, the "halo paradox" arose because of insufficient development of the earlier theory. Study of the distribution of apparent density in the halo in the simplest model constructed by us leads to the following conclusion: halos are observed at small values of λ (0.1-0.3), and at this time it is difficult to detect a displacement of the center (equal to $\frac{1}{2}\lambda R$), amounting to only 5-15% of the halo radius. Large relative errors in measuring the dimensions and position of the halo may exceed the small magnitude of the displacement of the center.

Leningrad Agricultural
Institute

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.