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Abstract

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STUDY OF PLASTIC DEFORMATION OF A METAL WITH DEFORMATION ANISOTROPY CREATED IN THE PROCESS OF PRELIMINARY LOADING

(Presented by Academician Yu. N. Rabotnov on 10 IV 1961)

In the present work we investigated the development of plastic deformations at the initial stages of secondary loading of a material partially or completely unloaded after primary plastic deformation.

To determine the deformations along a loading path it is necessary to study the relation between the vector of increments of plastic deformations $\delta\vec{\mathcal{E}}$ and the vector of increments of stresses $\delta\vec{S}$, depending on the loading history. It has been shown theoretically ([1]) that, for a smooth yield surface, the direction of the vector $\delta\vec{\mathcal{E}}$ coincides with the direction of the corresponding normal to this surface, while its magnitude is determined only by the magnitude of the projection $\delta S_{\delta\mathcal{E}}$ of the vector $\delta\vec{S}$ on $\delta\vec{\mathcal{E}}$. Thus, for a smooth yield surface the direction of the vector $\delta\vec{\mathcal{E}}$ and the value of the ratio $\delta\mathcal{E}/\delta S$ must depend only on the preceding deformation history, and therefore are not connected with the direction of the subsequent loading path. This proposition is fairly well confirmed experimentally as well (see below). The foregoing permits the relation between stresses and plastic deformations to be represented in the form

$$\delta\vec{\mathcal{E}} = h\,\delta\vec{S}_{\delta\mathcal{E}} \tag{1}$$

where h is a coefficient, hereafter called the modulus of plastic compliance. Practical application of formula (1) requires knowledge of the direction of the vector $\delta\vec{\mathcal{E}}$ and the value of h at each point of the loading path. At the same time, use of the proposition concerning the normality of the vector $\delta\vec{\mathcal{E}}$ to the yield surface is impracticable, owing to the impossibility of experimentally determining such a surface, since it must correspond to infinitely small increments of plastic deformations. The question of the possibility of using, for determining the direction of the vector $\delta\vec{\mathcal{E}}$, loci of stress states corresponding to the development of certain finite plastic deformations (corresponding to a certain tolerance for the intensity of increments of plastic deformations $\Delta\varepsilon_i$) has only been touched upon ([2]), and the form, dimensions, and position of the indicated loci have not

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yet been sufficiently studied. In the known literature there are no works at all devoted to a more or less systematic study of the modulus of plastic compliance.

In the present work, in a volume larger than in ([2]), loci constructed according to a tolerance for $\Delta\varepsilon_i$ were studied, and the possibility of using them to determine the directions of the vectors $\delta\vec{\mathcal{E}}$ was evaluated. At the same time, a study was undertaken of the regularities determining the magnitude of the modulus of plastic compliance, and the use of loci of equal h for describing the process of plastic deformation was proposed for the first time. The experiments were carried out under conditions of simultaneous tension and torsion of thin-walled tubular specimens made of technically pure nickel, which in the annealed state possessed a sufficiently high degree of homogeneity and isotropy. For the annealed material, the diagram—

plastic-deformation diagrams in the coordinates σ_i–ε_i under proportional loadings up to stresses of the order of 12 kg/mm^2 are satisfactorily approximated by the power-law relation

$$\varepsilon_i = a\sigma_i^k. \tag{2}$$

The material tested is characterized in greater detail in [3]. The apparatus used in the experiments was analogous to that described in [4]. The experimental data were processed using the representation of the deviators of the true stresses and true plastic strains in the form of vectors with components σ, $\sqrt{3}\tau$ and ε, $\frac{1}{\sqrt{3}}\gamma$.

The experimental investigation was begun by checking the assumption stated above that the direction of the vector $\overrightarrow{\delta\vartheta}$ and the magnitude h are independent of the direction of the vector $\overrightarrow{\delta S}$. For this purpose, 6 batches of specimens were tested (2–3 specimens in each batch). The specimens of each batch were subjected to the same primary loading, unloading, and secondary loading along a common path up to a certain prescribed point; upon reaching this point the loading paths of these specimens diverged. On the portion immediately following the branching point of the paths, the directions of the vector $\overrightarrow{\delta\vartheta}$ and the values of h were determined. Such experiments showed that, when the direction of the vector $\overrightarrow{\delta S}$ is changed, the vector $\overrightarrow{\delta\vartheta}$ deviates somewhat in the same direction; however, these fluctuations in the direction of the vector $\overrightarrow{\delta\vartheta}$ are very small ($\pm2^\circ$ from the mean position). The discrepancies in the values of h are likewise relatively small (up to $\pm5\div6\%$, on average) and are not systematic in character. Thus the experiments, to a sufficient approximation, confirmed the correctness of the assumptions underlying formula (1).

As was indicated, for practical use of formula (1) it is necessary to know the regularities that make it possible, at each point of the loading path, to determine the magnitude h and the direction of the vector $\overrightarrow{\delta\vartheta}$ as functions of the primary-loading path, the character of the secondary-loading path, and the stress state attained along this path.

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Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

In studying the influence of differences in secondary-loading paths, specimens were used whose primary-loading paths coincided, while the secondary-loading paths intersected at a specified point. Such experiments were repeated for 9 different cases of primary loading. Comparison of the directions of the vectors $\overrightarrow{\delta\vartheta}$, of the values of the plastic-compliance modulus h, and of the intensities of plastic strains under secondary loading $\Delta\varepsilon_i$ corresponding to the attainment of a specified stress state by different secondary paths showed that, in the majority of cases, the discrepancies are relatively small ($3 \div 4^\circ$ in the directions of the vectors $\overrightarrow{\delta\vartheta}$ and $\pm 5 \div 6\%$ in the values of $\Delta\varepsilon_i$ and h) and are of the same order as the discrepancies found in the experiments studying the influence of the direction of $\overrightarrow{\delta S}$.

The foregoing made it possible to conclude that, to a first approximation, the direction of $\overrightarrow{\delta\vartheta}$ and the quantities $\Delta\varepsilon_i$ and h are determined uniquely only by the stress state attained and by the history of the primary loading.

In studying the influence of the history of the primary loading, 16 different cases were investigated in which the primary plastic deformation was effected by linear tension (8 cases with different values of ε_{i0} from 0.2 to 10%), torsion (3 cases, $\varepsilon_{i0} = 0.2;\ 0.8$ and 1.4%), simultaneous tension and torsion with $\sigma = \sqrt{3}\tau$ (3 cases with the same values of ε_{i0}), and also by successive loading in the direct and reverse order by a tensile force and a torque ($\varepsilon_{i0} = 0.9\%$ in both cases). To obtain more complete information on the character of the anisotropy that developed as a result of the primary loading, for each

from the indicated paths a batch of 4–12 specimens was loaded; after being held under the full load, sufficient to stop noticeable creep, they were partially or completely unloaded and then loaded a second time along paths differing for each specimen. The secondary loading was carried out in steps. At each step the intensity of the stress increments was 0.5–1 kg/mm^2; passage through a step took

Fig. 1

Fig. 2

about 2 min. After each step, the increments of the strain components, the magnitude and direction of the vector $\delta\vec{\mathcal{E}}$, the value of the average modulus h for the given step, and the intensity $\Delta\varepsilon_i$ of the secondary plastic strains were determined. From the results of testing batches of specimens, on the stress plane the geometric loci of equal $\Delta\varepsilon_i$ and the geometric loci of equal h were

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Fig. 3

Figure 3: Fig. 3

constructed. The directions of the vectors $\delta\vec{\mathcal{E}}$ were compared with the directions of the normals to the indicated geometric loci.

Fig. 3

Some results of the processing described are presented in Figs. 1-3. In Fig. 1 the geometric loci of equal $\Delta\varepsilon_i$, constructed for a tolerance of 0.018%, are given for cases of primary loading along the path $\sigma = \sqrt{3}\tau$ up to $\sigma_{i0} = 15$ kg/mm^2 and along complex paths ending at the same values of σ_0 and τ_0; the numbering of the curves corresponds to the numbering of the primary-loading paths shown in the same figure. In Fig. 2 the geometric loci of equal h, constructed for cases of primary loading by a tensile force, are given. I and II are groups of curves corresponding to values of σ_{i0} equal, respectively, to 10.3 and 26.7 kg/mm^2. Curves a were obtained for

$$h = 10^4 \text{ mm}^2/\text{kg},$$

curves b for $h = 2 \cdot 10^{-4}$ mm^2/kg, and curves v correspond to

$$h = 10^{-3} \text{ mm}^2/\text{kg}.$$

In Fig. 3 the geometric loci of equal h ($h = 10^{-4}$ mm^2/kg) are compared for cases of primary loading along complex paths (the curve designations are the same as in Fig. 1). In the figures, at a number of points the directions of the vectors $\delta\vec{\mathcal{E}}$ are indicated by arrows. For other cases of primary loading or other values of $\Delta\varepsilon_i$ and h, the corresponding geometric

places, as well as the disposition of the vectors $\overrightarrow{\delta\mathcal{E}}$ with respect to these loci, have a character analogous to that shown in Figs. 1-3.

Analysis of the results made it possible to establish the following:

1. The directions of the vectors $\overrightarrow{\delta\mathcal{E}}$, as a rule, are determined with practically sufficient accuracy by the directions of the normals to the loci of equal $\Delta\varepsilon_i$. Deviations from normality increase somewhat as one moves away from the point at which the primary loading ends. Over a considerable length the loci of equal $\Delta\varepsilon_i$ are close to circles; however, especially in the case of primary loading along a complex path, they cannot throughout their entire extent be described by more or less simple equations, which makes practical use of the indicated loci difficult.

2. The loci of equal h are very close to circles. The centers of these circles are displaced relative to the origin of coordinates. The direction of the

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vector $\vec{\rho}$ of the displacement of the center is approximately determined by the ratio of the components of the primary plastic deformation, while the modulus of the vector $\vec{\rho}$ depends only on the value of h corresponding to the given circle and does not depend on the loading history. The radii R of the circles do not depend on the character of the path of primary loading and are determined only by the corresponding value of h and by the stress intensity σ_{i0} to which the primary loading was brought. On the basis of experimental data, the dimensions and position of the circles of equal h can be determined from the equations

$$\rho_\sigma = \frac{A}{h}\frac{\varepsilon_0}{\varepsilon_{i0}}, \qquad \rho_{\sqrt{3}\tau} = \frac{A}{h}\frac{\gamma_0}{\sqrt{3}\varepsilon_{i0}}; \tag{3}$$

$$h = ak\left[\frac{R+B}{\sigma_{i0}+B}D - B\right]^{k-1}, \tag{4}$$

where ρ_σ and $\rho_{\sqrt{3}\tau}$ are the corresponding projections of the vector $\vec{\rho}$; a, k, A, B, D are constants, of which a and k are determined from an experiment with any primary proportional loading (see formula (2)), and A, B, D from two experiments with secondary loading. The deviations of the experimental points from the circles constructed according to equations (3), (4) do not exceed $\pm 4\%$.

3. The directions of the vectors $\overrightarrow{\delta\mathcal{E}}$ systematically deviate from the directions of the normals to the circles of equal h. However, these deviations (on average 6-7°) are only slightly greater than the changes in the directions of the vectors $\overrightarrow{\delta\mathcal{E}}$ caused by the influence of the character of the secondary path, and in most cases they may be neglected.

4. To calculate the expected plastic deformations by formula (1), it is sufficient to have only the family of circles of equal h, whose construction according to equations (3), (4) requires knowledge of the five constants mentioned and of the stressed-deformed state of the material after primary loading.

Check experiments, whose results were compared with the results of numerical integration of formula (1), carried out using the circles of equal h constructed according to equations (3), (4), and the assumption of normality of the vectors $\overrightarrow{\delta\mathcal{E}}$ to these circles, showed that the components of the plastic deformations expected along a prescribed path of secondary loading can be predicted with an accuracy of up to 8-12%.

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