

# MOTION OF A PISTON WITH CONSTANT VELOCITY IN AN INHOMOGENEOUS MEDIUM

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**Abstract**

**Full Text**

**HYDROMECHANICS**

**M. P. MIKHAILOVA**

**MOTION OF A PISTON WITH CONSTANT VELOCITY IN AN INHOMOGENEOUS MEDIUM**

*(Presented by Academician L. I. Sedov, 10 VII 1961)*

Consider the motion of a gas behind a piston which expands with constant velocity  $u$  in a medium whose density  $\rho_n$  varies according to the law

$$\rho_n = \rho_1[1 - \varepsilon r^\chi], \quad (1)$$

where  $r$  is the linear coordinate;  $\varepsilon$  is a small parameter;  $\chi, \rho_1$  are constants. The unknown functions are the pressure  $p$ , the density  $\rho$ , and the velocity  $v$ , which depend on the variables  $t, r$  and on the parameters  $p_1, \rho_1, \varepsilon, \chi, \gamma$ , where  $\gamma = c_p/c_v$ . From these quantities one can form only two dimensionless variables

$$\lambda = \frac{\gamma p_1 t^2}{\rho_1 r^2}, \quad \mu = \varepsilon r^\chi.$$

Thus the sought dimensional functions can be represented through dimensionless functions depending on the dimensionless variables:

$$v = \frac{r}{t} \bar{V}(\lambda, \mu), \quad p = \rho_1 \left(\frac{r}{t}\right)^2 \bar{P}(\lambda, \mu), \quad \rho = \rho_1 \bar{R}(\lambda, \mu). \quad (2)$$

The problem of the motion of a gas behind a piston moving with constant velocity in a homogeneous medium was solved by L. I. Sedov <sup>(1)</sup>. Let  $V_0(\lambda)$ ,  $P_0(\lambda)$ ,  $R_0(\lambda)$  be the solutions of this problem. Then we represent the linearized sought solutions in the form

$$\bar{V} = V_0(\lambda) + \mu V(\lambda), \quad \bar{P} = P_0(\lambda) + \mu P(\lambda), \quad \bar{R} = R_0(\lambda) + \mu R(\lambda). \quad (3)$$

The basic equations of one-dimensional unsteady motion have the form

$$\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0,$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + (\nu - 1) \frac{\rho v}{r} = 0, \quad (4)$$

$$\frac{\partial}{\partial t} \left( \frac{p}{\rho^\gamma} \right) + v \frac{\partial}{\partial r} \left( \frac{p}{\rho^\gamma} \right) = 0.$$

Here  $\nu = 1, 2, 3$ , respectively, for plane, cylindrical, and spherical waves. After passing to dimensionless variables and varying with respect to  $\mu$ , we obtain a system of ordinary differential equations for  $V, P, R$ . From the independent variable  $\lambda$  we pass to the independent variable

variable  $V_0$ , and we shall seek solutions in the form

$$\begin{aligned} V &= (1 - V_0)^S \sum_{n=0}^{\infty} a_{ni} (1 - V_0)^n, \\ P &= (1 - V_0)^S \sum_{n=0}^{\infty} b_{ni} (1 - V_0)^n, \\ R &= (1 - V_0)^S \sum_{n=0}^{\infty} c_{ni} (1 - V_0)^n. \end{aligned} \quad (5)$$

As is known, in the case of self-similar motion the solutions for  $V_0, P_0, R_0$  are not expressed in analytic form. After passing to the independent variable  $V_0$ , approximate solutions can be found by expanding the desired functions in powers of  $(1 - V_0)$ :

$$\begin{aligned} \lambda &= \lambda_p \left[ 1 - \frac{2}{\nu} (1 - V_0) + \dots \right], \\ P_0 &= P_{0p} \left[ 1 - \frac{2}{\nu} (1 - V_0) + \dots \right], \\ R_0 &= R_{0p} [1 + \chi (1 - V_0)^2 + \dots], \end{aligned} \quad (6)$$

where  $\lambda_p, P_{0p}, R_{0p}$  are the values of the functions at the piston.

The characteristic equation of the system, if the equalities (6) are taken into account, will be

$$S^2 \left( S - \frac{\chi}{\nu} \right) = 0. \quad (7)$$

The roots  $S_1 = S_2 = 0$  correspond to a solution with a logarithmic singularity (2).

Let us now consider the boundary conditions. At the piston  $\vartheta = u$ , and since  $V_0 = 1$ , then  $V(1) = 0$ . Ahead of the piston, at some distance, a shock wave is moving. We shall assume that the piston velocity is large, and take the conditions at the shock wave in the form

$$\vartheta_2 = \frac{2}{\gamma + 1}c, \quad \rho_2 = \frac{\gamma + 1}{\gamma - 1}\rho_H, \quad p_2 = \frac{2}{\gamma + 1}\rho_H c^2, \quad (8)$$

where  $c$  is the velocity of the shock wave.

The conditions at the shock wave, after passage to dimensionless variables and variation with respect to  $\mu$ , have the form

$$\begin{aligned} V_2 &= \left[ \frac{2(1 + \chi)(1 + \lambda^*)}{\gamma + 1} - V_0(\lambda^*) + 2\lambda^* \left( \frac{dV_0}{d\lambda} \right)_{\lambda=\lambda^*} \right] a - \frac{2\lambda^*}{\gamma + 1}, \\ P_2 &= \left[ 2\lambda^* \left( \frac{dP_0}{d\lambda} \right)_{\lambda=\lambda^*} - 2P_0(\lambda^*) + \frac{4(1 + \chi)}{\gamma + 1} \right] a - \frac{2}{\gamma + 1}, \\ R_2 &= \left[ 2\lambda^* \left( \frac{dR_0}{d\lambda} \right)_{\lambda=\lambda^*} + \frac{4(\gamma + 1)(1 + \chi)\lambda^*}{\gamma - 1} \right] a - \frac{\gamma + 1}{\gamma - 1}, \end{aligned} \quad (9)$$

where  $\lambda^*$  is the value of  $\lambda$  at the shock wave, and  $a$  is a constant.

The radius vector  $r_2$  at the shock wave is represented in the form

$$r_2 = r_{20}[1 + a\mu^*]. \quad (10)$$

The solutions for  $V, P, R$  take the following form if  $(1 - V_0)^2$  is neglected and  $\chi/\nu$  is not an integer:

$$\begin{aligned} V &= \frac{\chi}{\nu} \frac{b_{01}}{\gamma P_{0p}} (1 - V_0) \ln(1 - V_0) + \frac{2\chi c_{03}}{R_{0p}(\chi + \nu)} (1 - V_0)^{\chi/\nu + 1}, \\ P &= b_{01} \left\{ \left[ 1 - \frac{\chi + 2}{\nu} (1 - V_0) \right] \ln(1 - V_0) + 1 + \frac{4}{\nu} (1 - V_0) \right\}, \end{aligned} \quad (11)$$

$$R = \frac{R_{0p}}{\gamma P_{0p}} b_{01} \left\{ \left[ 1 - \frac{\chi}{\nu} (1 - V_0) \right] \ln(1 - V_0) + 1 + \frac{\nu}{\chi} + \frac{4}{\nu - \chi} (1 - V_0) \right\} + c_{03} (1 - V_0)^{\chi/\nu}.$$

In these formulas the condition at the piston has already been taken into account. The unknown constants  $b_{01}, c_{03}, a$  are determined from the conditions at the shock wave.

If  $\chi/\nu$  is an integer, then the solutions will have the form

$$V = \frac{\chi}{\nu} \frac{b_{01}}{\gamma P_{0p}} (1 - V_0) \ln(1 - V_0),$$

$$P = b_{01} \left\{ \left[ 1 - \frac{\chi + 2}{\nu} (1 - V_0) \right] \ln(1 - V_0) + 1 + \frac{4}{\nu} (1 - V_0) \right\}, \quad (12)$$

$$R = \left[ \frac{R_{0p}}{\gamma P_{0p}} b_{01} + c_{11} (1 - V_0) \right] \ln(1 - V_0) + \left( 1 + \frac{\nu}{\chi} \right) \frac{R_{0p}}{\gamma P_{0p}} b_{01} + c_{11} (1 - V_0)$$

Fig. 1

Fig. 2

Fig. 3

The values of  $V, P, R$  computed by us as functions of  $V_0$  for  $\chi = 1$  in the cases  $\nu = 1, 2, 3$  are presented in the form of graphs (Figs. 1, 2, 3).

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## CITED LITERATURE

1. L. I. **Sedov**, *Methods of Similarity and Dimensionality in Mechanics*, Moscow, 1957.
2. G. **Piaggio**, *Integration of Differential Equations*, 1933.

*Note: Figure translations are in progress. See original paper for figures.*

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