APPROXIMATE EQUATIONS OF THE COUPLED OSCILLATIONS OF WATER AND A VESSEL IN THE CHAMBERS OF TRANSPORT SHIP LIFTS AND LOCKS

1961

SovietRxiv

Abstract

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HYDRAULICS

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APPROXIMATE EQUATIONS OF THE COUPLED OSCILLATIONS OF WATER AND A VESSEL IN THE CHAMBERS OF TRANSPORT SHIP LIFTS AND LOCKS

(Presented by Academician P. Ya. Kochina on 15 February 1961)

1. Let us denote: l_k –the length of the chamber of the ship lift or lock; b_k –the width of the chamber (or approach canal) at the free surface of the water; ω_k –the cross-sectional area of the liquid in the free chamber (or canal); l_c –the length of the vessel; b_c –the beam of the vessel at the waterline; ω_c –the cross-sectional area of the submerged part of the vessel; W –the volumetric displacement of the vessel; ω –the cross-sectional area of the liquid; b –the width of the free surface of the liquid; x, y, z –a moving coordinate system rigidly connected with the navigable chamber (the x-axis is directed along the chamber, the z-axis upward, and the origin of coordinates is coincident with one of the chamber gates); u_x –the horizontal component of the velocity of motion of the liquid particles relative to the chamber; j_x, j_z –the horizontal and vertical components of the acceleration of motion of the chamber; q –the discharge entering the lock (or canal) chamber through the distribution system per unit length; ζ –the ordinate of the free surface of the liquid; η –the ordinate of the plane of flotation of the vessel; η_0 –the vertical displacement of the center of gravity of the vessel (relative to the position of rest); ψ –the angle of inclination of the vessel (the angle of rotation about the transverse-horizontal axis); S –the area of the plane of flotation (bounded by the waterline); H –the longitudinal metacentric height of the vessel; M –the mass of the vessel; I –the moment of inertia of the vessel about the central transverse axis.

2. Considering the problem of a ship lift and taking as a basis the system of assumptions of the theory of long waves, in paper (1) we arrived at the dynamic equation of motion of the liquid

$$-\left[j_x + (g + j_z)\frac{\partial \zeta}{\partial x}\right] = \frac{\partial u_x}{\partial t} \tag{1}$$

and at the continuity equation

$$b\frac{\partial\zeta}{\partial t} + b_c\frac{\partial\eta}{\partial t} = -\frac{\partial(u_x\omega)}{\partial x}. \tag{2}$$

Here the problem is considered with a more accurate allowance for the vertical and longitudinal oscillations of the vessel.

Since $\eta(x,t) = \eta_0(t) + (x - l_0)\psi(t)$, where l_0 is the abscissa of the center of gravity of the vessel, from (1) and (2), taking into account the assumption of small oscillations, we obtain

$$b\frac{\partial^2\zeta}{\partial t^2} + b_c\left[\frac{d^2\eta_0}{dt^2} + (x - l_0)\frac{d^2\psi}{dt^2}\right] = \frac{\partial}{\partial x}\left\{\omega\left[j_x + (g + j_z)\frac{\partial\zeta}{\partial x}\right]\right\}. \tag{3}$$

In addition to equation (3), which describes the oscillatory motion of the liquid, we also formulate two equations for the vertical and longitudinal

oscillations of the vessel:

$$\frac{M + \Delta M}{\gamma'}\,\frac{d^2\eta_0}{dt^2} + S\eta_0 = \int_S \xi\, dS, \tag{4}$$

$$\frac{I + \Delta I}{\gamma'}\,\frac{d^2\psi}{dt^2} + WH\psi = \int_S (x - l_0)\,\xi\, dS, \tag{5}$$

where ΔM and ΔI are the added mass of water during vertical oscillations of the vessel and the added moment of inertia of the mass of water,

$$\gamma' = \left(1 + \frac{j_z}{g}\right)\gamma.$$

In calculating the force action of the liquid on the vessel, the assumption is made that the vessel has no influence on the pressure distribution over the transverse section of the liquid; i.e., the pressure distribution is assumed to occur in each section according to the hydrostatic law, as follows from the theory of long waves. However, this assumption excludes from consideration the influence of hydrodynamic forces—inertial and damping forces. The inertial forces are taken into account by introducing added masses. The damping forces may be omitted, since surface waves induced by the body cannot, in the bounded volume of liquid, carry away the energy expended on their formation. As is known from the theory of ship rolling, in vertical and pitching motion the frictional and vortex resistance may be regarded as negligibly small; we omit them as well.

Equations (3)–(5) form a closed system with the unknowns ξ, η_0, and ψ.

To simplify the problem we shall assume that the transverse section of the submerged part of the vessel does not vary along its length, i.e., has a cylindrical form*, whence $\omega_c = \text{const}$, $b_c = \text{const}$. Then, by virtue of the constancy of the transverse section of the chamber itself, $\omega = \omega_k - \omega_c$, $b = b_k - b_c$ will be piecewise constant functions of x. The system of equations (3)–(5) will take the form:

$$\frac{\partial}{\partial x}\left(\omega \frac{\partial \xi}{\partial x}\right) - \frac{b}{g'}\frac{\partial^2 \xi}{\partial t^2} = f(x,t); \tag{6}$$

$$\frac{M'}{\gamma'}\frac{d^2\eta_0}{dt^2} + S\eta_0 = b_c \int_{l_1}^{l_2} \xi \, dx; \tag{7}$$

$$\frac{I'}{\gamma'}\frac{d^2\psi}{dt^2} + WH\psi = b_c \int_{l_1}^{l_2} (x - l_0)\, \xi \, dx, \tag{8}$$

where it is denoted that

$$f(x,t) = \begin{cases} 0, & 0 \le x < l_1, \\ \dfrac{b_c}{g'}\left[\eta_0''(t) + (x - l_0)\psi''(t)\right], & l_1 < x < l_2, \\ 0, & l_2 < x \le l_k, \end{cases}$$

$$g' = g + j_z, \qquad M' = M + \Delta M, \qquad I' = I + \Delta I.$$

The boundary conditions in the end sections of the chamber are

$$j_x + g'\frac{\partial \xi(0,t)}{\partial x} = 0, \qquad j_x + g'\frac{\partial \xi(l_k,t)}{\partial x} = 0. \tag{9}$$

We shall denote ξ, ω, and b on the interval $(0, l_1)$ by ξ_1, ω_1, and b_1; on the interval (l_1, l_2) by ξ_2, ω_2, and b_2; and on the interval (l_2, l_k) by ξ_3, ω_3, and b_3.

* For large and medium river vessels this assumption is close to reality.

We shall write the matching conditions in the intermediate sections in the form

$$\zeta_i(l_i,t) = \zeta_{i+1}(l_i,t), \qquad \omega_i\left[j_x + g'\frac{\partial \zeta_i(l_i,t)}{\partial x}\right] = \omega_{i+1}\left[j_x + g'\frac{\partial \zeta_{i+1}(l,t)}{\partial x}\right] \quad (i = 1, 2). \tag{10}$$

If at the initial instant of time the liquid and the vessel are in a state of relative rest, the initial conditions will be

$$\zeta(x,0) = 0, \qquad \frac{\partial \zeta(x,0)}{\partial t} = 0; \tag{11}$$

$$\eta_0(0) = 0, \qquad \frac{d\eta_0(0)}{dt} = 0; \tag{12}$$

$$\psi(0) = 0, \qquad \frac{d\psi(0)}{dt} = 0. \tag{13}$$

The system of equations obtained characterizes the coupling between the oscillations of the liquid and the oscillations of the vessel. Equation (3) describes the oscillations of the liquid, but with allowance for the oscillations of the vessel, since it includes η_0 and ψ. Equations (4), (5) describe the forced oscillations of the vessel, the exciting forces being determined by the oscillations of the water surface (ζ).

The accelerated motion of the chamber directly causes oscillations of the liquid. These, in turn, excite oscillations of the vessel. But the oscillations of the liquid do not remain independent of the oscillations of the vessel; rather, they experience the reverse and very substantial influence of the latter. Thus, in the problem under consideration we are dealing with a typical case of coupled oscillations.

3. In a similar way one can refine the approximate formulation of the problem of oscillatory processes in the chambers and approach canals of locks, given by the author in the paper ([2]). Analogously to the preceding case, a system of the basic equations of the problem is derived which can be used in analyzing the conditions of vessel standing in lock chambers for the practically most important initial stage of filling the chamber, and, in analyzing the conditions of vessel standing and level oscillations in the approach canals, throughout the entire period of locking. The equations are suitable both for a distributed and for a concentrated system of lock supply. In contrast to the work of A. V. Mikhailov ([3]), in deriving the equations of water oscillations the theory of long waves is applied in simplified form, using linearized equations, but the oscillations of the vessel are taken into account much more accurately.

As applied to conditions in locks, instead of equation (3) we have

$$b\frac{\partial^2\zeta}{\partial t^2} + b_c\left[\frac{d^2\eta_0}{dt^2} + (x - l_0)\frac{d^2\psi}{dt^2}\right] = g\frac{\partial}{\partial x}\left(\omega\frac{\partial\zeta}{\partial x}\right) + \frac{\partial q}{\partial t}. \tag{14}$$

The equations of vessel oscillations (4) and (5) remain in force ($\gamma' = \gamma$). The boundary conditions, matching conditions, and initial conditions are formulated analogously to the preceding case.

4. In solving equations (4) and (5), it is necessary to know the magnitude of the added mass of liquid for a vessel located in the chamber of a navigation structure. This problem is important for the analysis of vertical and longitudinal oscillations of vessels both in ship lifts and in locks. The

transverse sections of the vessel and the chamber may be regarded as practically close to rectangular. An approximate solution was carried out by the author on the basis of the hypothesis of plane flow, the oscillations of the vessel frame being taken as vertical and small. The velocity potential was determined under an approximate condition on the free surface: it is assumed that the free surface oscillates while remaining horizontal across the width. This is justified by the smallness of the frequency of oscillations

the vessel, and also of the width of the free surface in comparison with the vessel' s draft. Thus the problem is reduced to the Neumann problem for a domain having a polygonal shape.

To determine the significance of the added mass, two positions of the vessel across the width of the chamber are of interest: (a) when it is pressed against the side of the chamber, and (b) when it is located along the axis of the chamber. Other conditions being equal, in the first case we have the largest value of the added mass, and in the second, the smallest. To integrate the Laplace equation in these cases, Schwarz' s algorithm was used[1]. Numerical calculations show that already the second approximation practically coincides with the first.

For an arbitrary position of the vessel across the width of the chamber, the grid method may be used (such calculations were carried out by V. M. Lavrent' eva).

It follows from the numerical calculations that the inertia of the vessel itself during its oscillations in the chambers of navigation structures proves to be insignificant in comparison with the inertia of the fluid set into oscillation.

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Received
6 II 1961

CITED LITERATURE

Note: Figure translations are in progress. See original paper for figures.

[1]L. V. Kantorovich, V. I. Krylov, *Approximate Methods of Higher Analysis*, Moscow-Leningrad, 1952.