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Abstract

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ON EXTENSIONS OF THE INDEXIAL OF A FINITE GROUP

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In papers ([1–5]) we proposed and developed a method of indexials for finding subgroups of a finite group. The essence of this device is that to every divisor h of the order of a finite group, in the case when h is a so-called indexial of this group, it is possible to associate at least one subgroup $\mathfrak{H}$ of order hc, where c is a certain multiplier for which $\Pi(c) \subseteq \Pi(h)$ (by $\Pi(n)$ we denote the set of all distinct prime divisors of the natural number n). At the same time we established some other properties of the subgroup $\mathfrak{H}$, specifying the properties of the additional multiplier c.

The criterion we obtained for the existence of subgroups included, as its special cases, many of the principal previously known results in this area (Sylow, P. Hall, Schur, etc.–see, for example, ([2])).

The subject of the present paper is a further investigation of the additional multiplier c in the theorem obtained earlier by us, with the aim of finding an estimate for the number of its distinct prime divisors. In doing so we shall use the definitions and notation introduced in ([2,4]). In particular, we shall regard the subgroup $\mathfrak{H}$ under investigation as a suitable subgroup of some normal, specially arithmetically closed (s.a.c.) extension ([4]) of a given indexial h.

2. Definition 1. An extension $(ch)_{R,\varphi}$ of the indexial $(h)_{R,f}$ will be called a **primary** s.a.c. extension if $(ch)_{R,\varphi}$ is an s.a.c. extension of the indexial $(h)_{R,f}$ and if among the coefficients $c_\beta, c_{\beta+1}, \dots, c_\omega$ there is one and only one greater than unity, which in this case will be a power of some prime number.

Definition 2. A finite sequence of $t+1$ indexials

$$(h)_{R,f} = (h^{(0)})_{R,f},\ (h^{(1)})_{R,f_1},\ \dots,\ (h^{(t)})_{R,f_t},$$

in which, for $t > 0$, each subsequent term is an s.a.c. extension of each preceding one and a primary s.a.c. extension of the immediately preceding term, will be called an **s.a.c. chain of length t issuing from the indexial** $(h)_{R,f}$. The greatest of the lengths of all s.a.c. chains issuing from the indexial $(h)_{R,f}$ will be called the **immersion of the indexial** $(h)_{R,f}$ and denoted by $\mathrm{imm}(h)_{R,f}$. If the length of some s.a.c. chain issuing from $(h)_{R,f}$ is equal to $\text{imm}(h)_{R,f}$, then such a chain will be called an **immersion chain of the indexial** $(h)_{R,f}$.

Definition 3. If n is a natural number, then by $N(n)$ we shall denote the number of distinct prime divisors of n. In particular, $N(1) = 0$.

Theorem. Every indexial $(h)_{R,f}$ of a finite group $\mathfrak{G}$ has at least one proper c.a.e. extension $(ch)_{R,\varphi}$, for which $N(c) \geq \text{imm}(h)_{R,f}$.

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Note: Figure translations are in progress. See original paper for figures.