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Abstract

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INTERACTION OF WEAK PRESSURE WAVES WITH A FLAME FRONT

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The interaction of pressure waves with a flame front has been investigated experimentally in works ([1],[2]) and theoretically in works ([3]–[7],[10]). Experiments show that the interaction of a shock wave with a flame front can lead to amplification of the shock wave. In works ([6],[7]) the coefficient of amplification of a shock wave was found for the case of relaxation interaction of a shock wave with a flame front. In work ([10]) a possible mechanism of amplification of a weak shock wave in a turbulent combustion zone was considered. Below we present a gas-dynamic treatment of the nonrelaxation interaction of weak pressure waves with a flame front, taking into account the change in the flame propagation velocity when the thermodynamic parameters of the combustible mixture change in a weak wave.

Fig. 1. 1—flame front, 2—entropy wave, 3—pressure wave.

Let us consider the case in which a weak pressure wave overtakes a flame front (see Fig. 1a). After interaction with the flame, a reflected pressure wave and an entropy wave will travel through the combustion products, while a transmitted pressure wave will propagate through the combustible mixture (Fig. 1b). These disturbances on both sides of the flame front are related by equations obtained by varying the conditions expressing the laws of conservation of mass, momentum, and energy at the flame front. The equations of conservation of mass, momentum, and energy at the flame front have the form

$$\rho_1 U = \rho_2(u_1 - u_2 + U); \qquad p_1 + \rho_1 U^2 = p_2 + \rho_2(u_1 - u_2 + U)^2;$$
$$w_1 + U^2/2 = w_2 + (u_1 - u_2 + U)^2/2. \tag{1}$$

Varying and retaining in the coefficients of the variation terms of zeroth and first order of smallness in U/c_1, where U is the flame propagation velocity and c_1 is the speed of sound, we write these equations in the form

$$\rho_1 \delta U + U \delta\rho_1^{\text{pr}} = \rho_2(\delta u_1^{\text{pr}} - \delta u_2^{\text{pad}} - \delta u_2^{\text{otr}} + \delta U) + U\frac{c_2^2}{c_1^2}(\delta\rho_2^{\text{pad}} + \delta\rho_2^{\text{otr}} + \delta\rho_2^{\text{e}});$$

$$\delta p_1^{\text{pr}} + 2\rho_1 U\delta U = \delta p_2^{\text{pad}} + \delta p_2^{\text{otr}} + 2\rho_2 U\frac{c_2^2}{c_1^2}(\delta u_1^{\text{pr}} - \delta u_2^{\text{pad}} - \delta u_2^{\text{otr}} + \delta U); \qquad (2)$$

$$\delta w_1^{\text{pr}} + U\delta U = \delta w_2^{\text{pad}} + \delta w_2^{\text{otr}} + \delta w_2^{\text{e}} + U\frac{c_2^2}{c_1^2}(\delta u_1^{\text{pr}} - \delta u_2^{\text{pad}} - \delta u_2^{\text{otr}} + \delta U).$$

In addition, the following relations hold on the waves: for the incident pressure wave,

$$\delta S_2^{\text{pad}} = 0; \qquad \delta u_2^{\text{pad}} = \frac{\delta p_2^{\text{pad}}}{\rho_2 c_2}; \qquad \delta w_2^{\text{pad}} = \frac{\delta p_2^{\text{pad}}}{\rho_2}; \qquad \delta \rho_2^{\text{pad}} = \frac{\delta p_2^{\text{pad}}}{c_2^2}; \quad (3)$$

reflected pressure wave

$$\delta S_2^{\text{refl}} = 0; \qquad \delta u_2^{\text{refl}} = -\frac{\delta p_2^{\text{refl}}}{\rho_2 c_2}; \qquad \delta w_2^{\text{refl}} = \frac{\delta p_2^{\text{refl}}}{\rho_2}; \qquad \delta \rho_2^{\text{refl}} = \frac{\delta p_2^{\text{refl}}}{c_2^2}; \quad (4)$$

transmitted pressure wave

$$\delta S_1^{\text{tr}} = 0; \qquad \delta u_1^{\text{tr}} = \frac{\delta p_1^{\text{tr}}}{\rho_1 c_1}; \qquad \delta w_1^{\text{tr}} = \frac{\delta p_1^{\text{tr}}}{\rho_1}; \qquad \delta \rho_1^{\text{tr}} = \frac{\delta p_1^{\text{tr}}}{c_1^2}; \qquad (5)$$

entropy wave

$$\delta p_2^{\text{e}} = 0; \qquad \delta u_2^{\text{e}} = 0; \qquad \delta w_2^{\text{e}} = T_2 \delta S_2^{\text{e}} = -\frac{c_2^2}{\rho_2(\gamma - 1)}\,\delta\rho_2^{\text{e}}. \qquad (6)$$

We find the variation of the flame-propagation velocity δU, taking the known dependence $U = f(\rho_1, T_1)$:

$$\delta U = \left(\frac{\partial f}{\partial \rho_1}\right)_{T_1}\delta\rho_1 + \left(\frac{\partial f}{\partial T_1}\right)_{\rho_1}\delta T_1 = A\,\delta\rho_1; \qquad A = \left(\frac{\partial f}{\partial \rho_1}\right)_{T_1} + \frac{\gamma - 1}{\gamma}\frac{T_1}{\rho_1}\left(\frac{\partial f}{\partial T_1}\right)_{\rho_1}. \qquad (7)$$

Substituting relations (3)–(7) into the system of equations (2) and eliminating from it $\delta\rho_2^{\text{e}}$, we find

$$\left[(\rho_1 - \rho_2)A + \frac{U}{c_1^2} + (\gamma - 1)\frac{U}{c_2^2} - \frac{\rho_2}{\rho_1 c_1}\right]\delta p_1^{\text{tr}} = \gamma\frac{U}{c_1^2}(\delta p_2^{\text{inc}} + \delta p_2^{\text{refl}}) - \frac{1}{c_2}(\delta p_2^{\text{inc}} - \delta p_2^{\text{refl}}); \qquad (8)$$

$$\delta p_1^{\mathrm{tr}} = \left(1 + 2\frac{U}{c_1}\right)(\delta p_2^{\mathrm{inc}} + \delta p_2^{\mathrm{refl}}) - 2\frac{Uc_2}{c_1^2}(\delta p_2^{\mathrm{inc}} - \delta p_2^{\mathrm{refl}}).$$

Hence, for the magnitude of the acoustic conductivity of the flame front, we obtain

$$\zeta = \frac{\rho_2 c_2}{\rho_1 c_1} - (\rho_1 - \rho_2)c_2 A + (\gamma - 1)\left(1 - \frac{c_1^2}{c_2^2}\right)\frac{Uc_2}{c_1^2} +$$
$$+ 2(\rho_1 - \rho_2)A\left[c_1 - (\rho_1 - \rho_2)Ac_2^2\right]\frac{Uc_2}{c_1^2}. \qquad (9)$$

The reflection coefficient $k = \delta p_2^{\mathrm{refl}}/\delta p_2^{\mathrm{inc}}$ is related to the quantity ζ by the relation $k = (1-\zeta)/(1+\zeta)$. From the system (2) we also find the refraction coefficient $l = \delta p_1^{\mathrm{tr}}/\delta p_2^{\mathrm{inc}}$, which is equal to

$$l = \frac{2}{1+\zeta_0}\left[1 + 2(\rho_1 - \rho_2)AU\frac{c_2^2}{c_1^2} - \frac{B}{1+\zeta_0}\frac{U}{c_1}\right], \qquad (10)$$

where

$$\zeta_0 = \rho_2 c_2/\rho_1 c_1 - (\rho_1 - \rho_2)c_2 A;$$

$$B = \left\{(\gamma - 1)(1 - c_1^2/c_2^2) + 2(\rho_1 + \rho_2)A\left[c_1 - (\rho_1 - \rho_2)Ac_2^2\right]\right\}\frac{c_2^2}{c_1^2}.$$

It is seen from formula (9) that, in the first approximation, the acoustic conductivity of the flame front is a sum whose individual terms correspond to different physical factors. The first term is the acoustic conductivity of a contact discontinuity equivalent to the flame in the magnitude of the density jump. The second term, containing the physicochemical constant A, takes into account the influence of the flame reaction on weak disturbances. Since A, as a rule, is a positive quantity, the second term is negative and reduces the acoustic conductivity of the flame. Thus, already in the zeroth approximation, the effect caused by the change in the flame-propagation velocity in a weak wave is qualitatively manifested. With incre-

with increasing A, the amplitude of the reflected and transmitted waves increases. The third term, which is of first order of smallness, takes into account the entropy wave separating the combustion products formed before and after the moment of interaction. The fourth term takes into account the change in the flame-propagation velocity in the first approximation.

When the incident wave moves toward the flame front, the disturbance in the hot mixture is composed of the incident and reflected pressure waves, while in the combustion products it is composed of the transmitted pressure wave and the entropy wave; the variation of the flame-propagation velocity is related to the pressure disturbance by the relation $\delta U = A(\delta p_1^{\mathrm{inc}} + \delta p_1^{\mathrm{refl}})$. The formulas for the acoustic conductance and the refraction coefficient in this case have the form

$$\zeta' = \frac{\rho_1 c_1}{\rho_2 c_2} - (\rho_1 - \rho_2) c_2 A \frac{c_2}{c_1} + (\gamma - 1)\left(1 - \frac{c_1^2}{c_2^2}\right)\frac{U c_2}{c_2^2}\frac{c_2}{c_1} - 2(\rho_1 - \rho_2) A U \left(\frac{c_2}{c_1}\right)^3; \tag{11}$$

$$l' = \frac{1}{1+\zeta_0'}\left[1 - 2(\rho_1 - \rho_2) A U \frac{c_2^2}{c_1^2} - \frac{B'}{1+\zeta_0'}\frac{U}{c_1}\right], \tag{12}$$

where

$$\zeta_0' = \rho_1 c_1/\rho_2 c_2 - (\rho_1 - \rho_2) c_2 A c_2/c_1; \quad B' = \left[(\gamma - 1)\left(1 - c_1^2/c_2^2\right) - 2(\rho_1 - \rho_2) c_2 A\right] c_2^2/c_1^2.$$

Formula (11) shows that for $A = 0$, as in the preceding case, in the zeroth approximation the flame has an acoustic conductance equal to the acoustic conductance of the equivalent contact discontinuity. In contrast to case (9), where the reflected and transmitted waves had the same sign as the incident wave, formula (11) gives that, for sufficiently small A, the reflected wave has a sign opposite to that of the incident wave, while the sign of the transmitted wave coincides with the sign of the incident wave. In this case, as A increases, the amplitude of the reflected wave decreases. In other respects, the physical meaning of the individual terms in (11) is analogous to the physical meaning of the corresponding terms in (9). Of interest is the limiting case $\rho_1 \gg \rho_2$, corresponding to surface combustion of condensed fuel. From (9) it is seen that in this case the acoustic conductance of the combustion front is always negative and is equal to

$$\zeta_R = -(\rho_1 - \rho_2) c_2 A. \tag{13}$$

Consequently, amplification of the reflected wave takes place.

It should be noted that the applicability of formulas (9)–(13) is limited to not too large values of the constant A. Otherwise the flame, under the action of a weak wave, could generate waves of finite intensity, which goes beyond the limits of the linear treatment.

The constant A is the coefficient of the flame-propagation velocity under an adiabatic change of pressure in the combustible mixture. It can be found, for example, from the Zel' dovich—Frank-Kamenetskii formula for the flame-propagation velocity ([8]), or determined from experiment. In experiments one usually determines the dependence of the flame-propagation velocity on the initial temperature at constant pressure, or on pressure at constant temperature. The resulting coefficients of the flame-propagation velocity with respect to temperature and pressure under different conditions may be either positive or negative. Therefore the quantity A may be positive, negative, or zero.

As an example, consider combustion of a methane–air mixture. According to experimental data ([9]), in this case the dependence of the velocity on pressure is weak. Therefore it may be assumed that A is determined only by the dependence $U = U(T_r)$. For a mixture containing 10%

methane, at $T_0 = 20^\circ$, $P = 1$ atm.; $T_r = 2100^\circ$, $\Delta U/\Delta T = 0.2$ cm/sec · deg, and the individual terms in formula (9) are equal to:

$$\frac{\rho_2 c_2}{\rho_1 c_1} = 0.35; \quad (\rho_1 - \rho_2) c_2 A = 0.002; \quad (\gamma - 1)\left(1 - \frac{c_1^2}{c_2^2}\right)\frac{U c_2}{c_1^2} = 0.0009;$$

$$2(\rho_1 - \rho_2) c_2 A \left[\frac{c_1}{c_2} - (\rho_1 - \rho_2) c_2 A\right] \frac{c_2}{c_1}\frac{U}{c_1} = 3.5 \cdot 10^{-6}.$$

Thus, taking into account the change in the flame-propagation velocity during the interaction in the present case leads to a decrease in the acoustic conductivity of the flame front by 0.5%. Estimates show that in some cases this quantity may reach several percent. This effect may play a significant role if the pressure wave, owing to reflections, crosses the flame front many times.

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