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Abstract

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THEORY OF ELASTICITY

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STABILITY OF WING PANELS UNDER HEATING

(Presented by Academician Yu. N. Rabotnov on 29 VII 1960)

A thin-walled structure of the multi-spar wing type, under nonstationary aerodynamic heating, may lose stability owing to temperature stresses arising because the skin is heated more rapidly than the spar webs.

Let the structure have a large number of bays and sufficiently large dimensions in the direction of the y-axis, perpendicular to the axes x_1 and x_2 (Fig. 1). The initial temperature T_0 is the same at all points. Heating occurs through heat transfer to both outer surfaces from an external medium with temperature T_{B} and with a constant heat-transfer coefficient α^*. We shall neglect the nonuniformity of the temperature distribution through the thickness of the elements and the change, with temperature, of the thermophysical and elastic characteristics of the material.

The solution of the heat-conduction equations for elements 1 and 2

$$\frac{\partial\theta_1}{\partial\tau} = \frac{\partial^2\theta_1}{\partial\xi_1^2}, \qquad 0 \le \xi_1 \le 1, \tag{1}$$

$$\frac{\partial\theta_2}{\partial\tau} = \frac{\partial^2\theta_2}{\partial\xi_2^2} - h^2\theta_2, \qquad 0 \le \xi_2 \le \beta,$$

taking into account the boundary conditions, the symmetry conditions of the heating, and the initial condition

$$[\theta_1]_{\xi_1=0} = [\theta_2]_{\xi_2=0}, \qquad \left[\frac{\partial\theta_1}{\partial\xi_1}\right]_{\xi_1=0} = -R\left[\frac{\partial\theta_2}{\partial\xi_2}\right]_{\xi_2=0},$$

$$\left[\frac{\partial\theta_1}{\partial\xi_1}\right]_{\xi_1=1} = \left[\frac{\partial\theta_2}{\partial\xi_2}\right]_{\xi_2=\beta} = 0, \qquad [\theta_1]_{\tau=0} = [\theta_2]_{\tau=0} = 1$$

Fig. 1

Figure 1: Fig. 1

may be represented in the form of integrals

$$\theta_i(\xi_i,\tau)=\frac{1}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty}\theta_i^*(\xi_i,p)e^{p\tau}\,dp,\qquad \varepsilon>0,\ i=1,2, \tag{2}$$

where

$$\theta_1^*=\frac{1}{p}+A_1\operatorname{ch}r_1(1-\xi_1),\qquad \theta_2^*=\frac{1}{p+h^2}-A_2\operatorname{ch}r_2(\beta-\xi_2),$$

$$A_1=Rr_2\frac{S\ \operatorname{th}\beta r_2}{Q\ \operatorname{ch}\beta r_1},\qquad A_2=r_1\frac{S\ \operatorname{th}r_1}{Q\ \operatorname{ch}\beta r_2},\qquad S=\frac{1}{r_2^2}-\frac{1}{r_1^2},$$

$$Q=Rr_2\operatorname{th}\beta r_2+r_1\operatorname{th}r_1,\qquad r_1=\sqrt{p},\qquad r_2=\sqrt{p+h^2},$$

$$\theta_i=\frac{T_{\text{B}}-T_i}{T_{\text{B}}-T_0},\qquad h^2=\frac{\alpha^*}{\gamma_2c_2\delta_2}\frac{a^2}{k_1},\qquad \tau=t\frac{k_1}{a^2},\qquad \beta=\frac{b}{a}\sqrt{\frac{k_1}{k_2}},$$

$$R=\frac{2\delta_2\lambda_2}{\delta_1\lambda_1}\sqrt{\frac{k_1}{k_2}},\qquad \xi_1=\frac{x}{a},\qquad \xi_2=\frac{x_2}{b}\beta.$$

T_i is the temperature, λ_i the thermal conductivity, c_i the heat capacity, γ_i the specific weight, k_i the thermal diffusivity, δ_i the thickness; the index $i=1$ refers to the web, the index $i=2$ to the skin.

Taking into account self-equilibration in the direction of the y axis, for the temperature stresses in the skin we shall have

$$\sigma_{y2}=\alpha T_0E_2\left[\frac{1}{\beta(1+\varkappa)}\int_0^\beta\theta_2\,d\xi_2+\frac{\varkappa\psi}{1+\varkappa}\int_0^1\theta_1\,d\xi_1-\theta_2\right], \tag{3}$$

where

$$\varkappa=\frac{\delta_1}{2\delta_2}\frac{aE_1}{bE_2},\qquad \frac{\alpha_1}{\alpha_2}=\psi,$$

E_i is the modulus of elasticity, α_i is the coefficient of linear expansion.

Fig. 2

Figure 2: Fig. 2

Fig. 1. 1 —stringer, 2 —skin

In solving the stability problem, we shall regard the skin panel as a plate infinitely long in the direction of the y axis and hinged on the stringers. Assuming for the deflections

$$w(x_2, y) = \sin\frac{\pi x_2}{2b}\sin\frac{\pi y}{l},$$

and introducing the expressions for the stresses (3), with the representation of the temperatures in the form (2), into the stability equation in Galerkin form,

$$\int_0^b\int_0^l\left(D_2\Delta\Delta w - \sigma_{y2}\delta_2\frac{\partial^2 w}{\partial y^2}\right)w\,dx_2dy = 0,$$

then, changing the order of integration and evaluating the integrals with respect to ξ_1 and ξ_2, we obtain

Fig. 2. a —1st test, b —2nd test

$$2Z - \frac{\varkappa}{1+\varkappa}\left(1 - e^{-h^2\tau}\right) +$$

$$+\frac{h^2}{2\pi i}\int_{\varepsilon-i\infty}^{\varepsilon+i\infty}\frac{\operatorname{th} r_1\,\operatorname{th}\beta r_2}{Qr_1^2r_2^2}\left[\frac{\varkappa}{\beta(1+\varkappa)}\frac{r_1}{r_2} + \frac{\varkappa\psi R}{1+\varkappa}\frac{r_2}{r_1} - \frac{\beta}{\pi^2+\beta^2r_2^2}\,r_1r_2\right]e^{p\tau}\,dp = 0, \tag{4}$$

where

$$Z = \frac{\pi^2}{24(1-\mu_2^2)(T_{\text{B}} - T_0)\alpha_2}\left(\frac{\delta_2}{b}\right)^2.$$

Let us note that retaining in the expression for the deflection also the term

$$\sin\frac{3\pi x}{2b}\sin\frac{\pi y}{l}$$

refines the value of Z by an amount less than 1%.

Using the residue theorem, from (4) we obtain the final equation, containing only the parameters of the problem and the time:

Fig. 3

Figure 3: Fig. 3

$$Z - \frac{\varkappa(1-\psi)}{2(1+\varkappa)} - h^2 \sum_{j=1}^{\infty} \Omega_j e^{-\nu_j^2 \tau} = 0, \tag{5}$$

where

$$\Omega_j = \frac{\operatorname{tg}^2 \nu_j}{(h^2 - \nu_j^2)\Phi_j} \left[-\frac{\varkappa}{R\beta(1+\varkappa)(h^2-\nu_j^2)} + \frac{\varkappa\psi}{1+\varkappa}\frac{1}{\nu_j^2} + \frac{\beta}{R}\frac{1}{\pi^2 + \beta^2(h^2-\nu_j^2)} \right],$$

$$\Phi_j = R\beta - \frac{\beta}{R}\frac{\nu_j^2 \operatorname{tg}^2 \nu_j}{h^2 - \nu_j^2} + \sec^2 \nu_j + \frac{h^2 \operatorname{tg} \nu_j}{\nu_j(h^2 - \nu_j^2)}.$$

and ν_j are the roots of the equations

$$R\sqrt{h^2-\nu^2}\ \operatorname{th} \beta\sqrt{h^2-\nu^2} - \nu \operatorname{tg} \nu = 0, \qquad \nu < h,$$

$$R\sqrt{\nu^2-h^2}\ \operatorname{tg} \beta\sqrt{\nu^2-h^2} + \nu \operatorname{tg} \nu = 0, \qquad \nu > h.$$

Equation (5) determines the critical value of the time.

It is interesting to note that the solution of the stability problem has been obtained here by using integral representations for the temperatures and stresses, so that it was not necessary to calculate the temperatures and stresses themselves. Obviously, such a procedure is more effective than that proposed in papers ([1,2]).

Fig. 3

To each value of Z, according to equation (5), there correspond two, one, or no critical values of τ. In the case when there are two values of τ, the first of them corresponds to the moment of loss of stability, and the second to the moment when the initial form again becomes stable, provided that in the postcritical state the panel has not acquired residual deformations.

The relation between the parameter Z and the critical value of the time, calculated from equation (5) for the case $\beta = 3.68$, $R = 0.64$, $h = 1.0$, $\psi = 1.0$, is represented by the curve in Fig. 2.

An experimental check of the results obtained was carried out on a model of a four-spar wing with skin and webs made of the same material ($\psi = 1$), under heating in conditions of an instantaneous start. Strain gauges, oriented in the direction of the y-axis, were glued to the inner side of the skin in the central bay. An oscillogram of the sensor readings is shown in Fig. 3.

Because of the different positions of the sensors relative to the waves arising during buckling, the moments of loss of stability and return to the stable state are clearly recorded.

The results of the experimental determination of the critical time values for the case $Z = 0.0658$, $R = 0.64$, $\beta = 3.68$, $h = 1.0$ are shown in Fig. 2.

A. G. Zagorskii took part in carrying out the experiment.

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Note: Figure translations are in progress. See original paper for figures.