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Abstract

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APPLICATION OF THE COMPLETE SYSTEM OF EQUATIONS OF THERMO-HYDRODYNAMICS TO SHORT-RANGE WEATHER FORECASTING IN A TWO-LEVEL MODEL

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Recently, in order to improve short-range weather forecasting, special attention has been given to the application of the complete system of equations of thermo-hydrodynamics. The theoretical schemes developed by I. A. Kibel' ([1,2]) give a clear connection between the general and quasi-geostrophic theories, and also indicate wherein they differ. For the implementation of these schemes on a computing machine, it is useful at the first stage to simplify the problem somewhat. The present work is a scheme modified for a two-level model ([2]).

A simple derivation of the initial equations for a two-level model may be found, for example, in the work of A. Eliassen ([3]). It is sufficient to write two equations of motion and the continuity equation at the levels 250 and 750 mb, denoted below respectively by the indices 1 and 3, and the heat-inflow equation at the 500 mb level, denoted by the index 2, so that

$$\frac{\partial u_i}{\partial t} + \frac{\partial \Phi_i}{\partial x} - lv_i = -\frac{\partial u_i^2}{\partial x} - \frac{\partial u_i v_i}{\partial y} - \left(\frac{\partial u\omega}{\partial \xi}\right)_i, \qquad i = 1, 3; \tag{1}$$

$$\frac{\partial v_i}{\partial t} + \frac{\partial \Phi_i}{\partial y} + lu_i = -\frac{\partial u_i v_i}{\partial x} - \frac{\partial v_i^2}{\partial y} - \left(\frac{\partial v\omega}{\partial \xi}\right)_i, \qquad i = 1, 3; \tag{2}$$

$$\frac{\partial u_i}{\partial x} + \frac{\partial v_i}{\partial y} + \left(\frac{\partial \bar{\omega}}{\partial \xi}\right)_i = 0, \qquad i = 1, 3; \tag{3}$$

$$\left(\xi^2 \frac{\partial^2 \Phi}{\partial \xi\, \partial t}\right)_2 + c^2 \bar{\omega}_2 = -\left[\xi^2 \left(u_2 \frac{\partial^2 \Phi}{\partial x\, \partial \xi} + v_2 \frac{\partial^2 \Phi}{\partial y\, \partial \xi}\right)\right]_2. \tag{4}$$

(the notation is the same as in ([2])).

We shall consider the "plane problem" and the boundary conditions

$$\bar{\omega} = 0 \quad \text{for } \xi = 0,\ 1. \tag{5}$$

Replacing the derivatives with respect to ξ by finite differences, taking (5) into account, gives
$(\partial u\bar{\omega}/\partial\xi)_1 \simeq 2(u\bar{\omega})_2$,
$(\partial u\bar{\omega}/\partial\xi)_3 \simeq -2(u\bar{\omega})_2$,
$(\xi^2\partial\Phi/\partial\xi)_2 \simeq \frac{1}{2}(\Phi_3 - \Phi_1)$, and

$$2\bar{\omega}_2 \simeq -\left(\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y}\right) \simeq \left(\frac{\partial u_3}{\partial x} + \frac{\partial v_3}{\partial y}\right).$$

Introduce

$$U = \frac{1}{2}(u_1 + u_3), \qquad V = \frac{1}{2}(v_1 + v_3), \qquad H = \frac{1}{2}(\Phi_1 + \Phi_3),$$

$$u = \frac{1}{2}(u_1 - u_3), \qquad v = \frac{1}{2}(v_1 - v_3), \qquad h = \frac{1}{2}(\Phi_1 - \Phi_3);$$

then, with the aid of

of addition and subtraction, taking into account the replacement of u_2, v_2 by U, V, respectively, we obtain the system of equations for these 6 functions (3):

$$\begin{aligned}
&\frac{\partial U}{\partial t} + \frac{\partial H}{\partial x} - lV = -\left[\frac{\partial(U^2 + u^2)}{\partial x} + \frac{\partial(UV + uv)}{\partial y}\right] \equiv A_1,\\
&\frac{\partial V}{\partial t} + \frac{\partial H}{\partial y} + lU = -\left[\frac{\partial(UV + uv)}{\partial x} + \frac{\partial(V^2 + v^2)}{\partial y}\right] \equiv A_2,\\
&\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0;
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\frac{\partial u}{\partial t} + \frac{\partial h}{\partial x} - lv = -\left[\frac{\partial uU}{\partial x} + v\frac{\partial V}{\partial y} + V\frac{\partial u}{\partial y}\right] \equiv a_1,\\
&\frac{\partial v}{\partial t} + \frac{\partial h}{\partial y} + lu = -\left[\frac{\partial vV}{\partial y} + u\frac{\partial V}{\partial x} + U\frac{\partial v}{\partial x}\right] \equiv a_2,\\
&\frac{\partial h}{\partial t} + \frac{c^2}{2}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = -\left[U\frac{\partial h}{\partial x} + V\frac{\partial h}{\partial y}\right] \equiv a_3,
\end{aligned} \tag{7}$$

and $\overline{\omega}_2$ is found from the equation

$$\overline{\omega}_2 = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right). \tag{8}$$

These equations are solved over the entire plane with the prescribed initial values U^0, V^0, u^0, v^0, and h^0.

Let us replace the derivatives with respect to t by finite-difference relations, for example $\partial u/\partial t \approx (u - u^0)/\delta t$, etc., taking into account that in the left-hand sides we use finite differences centered with respect to t, while the right-hand sides are left arbitrary for the time being; for example, the equation for u takes the form

$$\frac{u - u^0}{\delta t} + \frac{1}{2}\left[\frac{\partial(h + h^0)}{\partial x} - l(v + v^0)\right] \equiv a_1.$$

After some transformations we obtain the prognostic equations

$$\Delta(U - U^0) = -\frac{\partial}{\partial y}\left[\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right) - \beta V\right]\delta t \equiv -\frac{\partial B_\Omega}{\partial y}\delta t; \tag{9}$$

$$\Delta(V - V^0) = \frac{\partial}{\partial x}\left[\left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right) - \beta V\right]\delta t \equiv -\frac{\partial B_\Omega}{\partial x}\delta t; \tag{10}$$

$$\Delta H = l\left(\frac{\partial V}{\partial x} - \frac{\partial U}{\partial y}\right) + \left[\left(\frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y}\right) - \beta U\right] \equiv F; \tag{11}$$

$$u = \frac{1}{1 + (\delta/2)^2}\left[a_1 \cdot \delta t + \frac{\delta}{2}a_2 \cdot \delta t + \left(1 - \left(\frac{\delta}{2}\right)^2\right)u^0 + \delta \cdot v^0 - \left(\frac{\delta}{2}\right)\frac{1}{l}\frac{\partial(h + h^0)}{\partial x} - \left(\frac{\delta}{2}\right)^2\frac{1}{l}\frac{\partial(h + h^0)}{\partial y}\right]; \tag{12}$$

$$v = \frac{1}{1 + (\delta/2)^2}\left[-\frac{\delta}{2}a_1 \cdot \delta t + a_2 \cdot \delta t - \delta \cdot u^0 + \left(1 - \left(\frac{\delta}{2}\right)^2\right)v^0 + \left(\frac{\delta}{2}\right)^2\frac{1}{l}\frac{\partial(h + h^0)}{\partial x} - \left(\frac{\delta}{2}\right)\frac{1}{l}\frac{\partial(h + h^0)}{\partial y}\right]; \tag{13}$$

$$\begin{aligned}\Delta(h - h^0) - \frac{l^2}{c^2/2}\left(1 + \frac{1}{(\delta/2)^2}\right)(h - h^0) = \delta \cdot \Bigg[&\left(\frac{\partial a_2}{\partial x} - \frac{\partial a_1}{\partial y}\right) \\ &-\frac{1}{c^2/2}\left(1 + \frac{1}{(\delta/2)^2}\right)la_3 - \beta\left(\frac{v + v^0}{2}\right)\Bigg] + \left[2\left(\frac{\partial a_1}{\partial x} + \frac{\partial a_2}{\partial y}\right)\right. \\ &\left.-\beta(u + u^0) + 2(l\Omega^0 - \Delta h^0)\right] + \frac{4}{\delta}lD^0 \equiv f,\end{aligned} \tag{14}$$

where $\Delta \equiv \partial^2/\partial x^2 + \partial^2/\partial y^2$, $\delta \equiv l \cdot \delta t$, $\Omega^0 \equiv \partial v^0/\partial x - \partial u^0/\partial y$, and $D^0 \equiv \partial u^0/\partial x + \partial v^0/\partial y$.

The solution of (14) with respect to $h - h^0$ has the form

$$h - h^0 = -\frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} \int K_0 \left(\frac{c}{\sqrt{2l}} \sqrt{1 + \left(\frac{2}{\delta}\right)^2} \sqrt{(x-x')^2 + (y-y')^2} \right) t(x', y')\, dx'\, dy', \tag{15}$$

where $K_0(x)$ is the Bessel function of imaginary argument.

In practice we may regard all nonlinear terms and the terms connected with β as having initial values; then from (15) one can find h, then from (12), (13) find u, v, and from (9), (10) find U, V; H and ω_2 are computed from the solution of (11) and (8) at the time of interest to us.

As for the solutions for U, V, and H, we adopt the "local solution" of the Poisson equation, taking into account that the wind divergence should be strictly equal to zero also when derivatives are replaced by finite differences, so that

$$U - U^0 = \left(-\frac{\partial}{\partial y} \int\limits_0^{2\pi} \int\limits_0^{R} \frac{\varepsilon}{2\pi} \ln \frac{R}{r} (-B_\Omega)\, r'\, dr'\, d\theta \right) \delta t; \tag{16}$$

$$V - V^0 = \left(\frac{\partial}{\partial x} \int\limits_0^{2\pi} \int\limits_0^{R} \frac{\varepsilon}{2\pi} \ln \frac{R}{r} (-B_\Omega)\, r'\, dr'\, d\theta \right) \delta t; \tag{17}$$

$$H(t) - H^0 = \int\limits_0^{2\pi} \int\limits_0^{R} \frac{\varepsilon}{2\pi} \ln \frac{R}{r} (F(t) - \Delta H^0)\, r'\, dr'\, d\theta, \tag{18}$$

where R and ε are determined empirically. We took $R = 900$ km, $\varepsilon = 1.5$.

This scheme was programmed by us for BESM-2. Several 24-hour forecasts were made with $\delta t = 30$ min., $\Delta x = \Delta y = 300$ km.

In conclusion I express my deep gratitude to Corresponding Member of the Academy of Sciences of the USSR I. A. Kibel for the attention he showed to my work.

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Note: Figure translations are in progress. See original paper for figures.