# Hydraulics

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## Abstract

## Full Text

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# TRANSIENT FLOW OF GASES IN MAIN PIPELINES

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To solve many problems connected with the transfer of gas in main pipelines, it is necessary to study transient flow regimes. Thus, for example, determining the time for pressure recovery at the terminal point of a pipeline after a valve is closed is of practical importance in problems of automating the transport of liquids. The study of transient regimes also makes it possible to construct practical methods, convenient for determining—by means of comparatively simple calculations—the hydrodynamic characteristics of a pipeline and the physical properties of the liquids being transported. Let us note, for example, the determination of the actual value of the friction coefficient, which in turbulent flow depends, in addition to the Reynolds number, also on the effective roughness of the pipeline. In practice such a determination encounters very great difficulties, since the roughness of pipelines changes with time owing to the corrosive and abrasive action of the transported liquids.

In hydrodynamic calculations for establishing the parameters that ensure the transport of liquids under conditions optimal from the energy standpoint, values of physical properties are introduced on the basis of analysis of very small samples.

Naturally, the values obtained in this way do not reflect the actual ones, since in main pipelines, generally speaking, inhomogeneous mixtures of gases are transported. With the aid of methods based on mathematical processing of the resulting measurement data obtained in pipelines during transient flow, one can establish the mean actual values of the physical properties corresponding to the entire length and, consequently, to the entire volume of the pipeline.

In the present work we shall consider the problem of the growth of dynamic pressure in a main gas pipeline after the inflow to its terminal point has been stopped.

If one considers the case of turbulent flow and takes for the velocity its value averaged over the cross section, then the unsteady flow, without taking into account the nonlinear inertial term, whose value may be neglected, is determined by the system of equations

$$-\frac{\partial p}{\partial x} = \rho\left(\frac{\partial v}{\partial t} + \frac{\lambda v^2}{2d}\right), \qquad -\frac{\partial p}{\partial t} = \frac{\partial(\rho v)}{\partial x}, \tag{1}$$

where $p$ is the pressure, $v$ is the velocity averaged over the cross section, $d$ is the diameter of the pipeline, and $\lambda$ is the hydraulic friction coefficient.

As shown in [1], for main pipelines whose lengths are of the order of hundreds of kilometers, the influence on the flow of the term $\partial v/\partial t$ is practically insignificant.

Integration of system (1), because of the nonlinearity of the equations, cannot be carried out by known methods, and, in order to obtain results convenient for application, the problem must be solved approximately. To linearize system (1) we shall use the method employed in [2], based on the assumption that, over some appropriately chosen interval of pressure variation, the equation of state of real gases may be replaced by an exponential function, while the quadratic term, with allowance for friction, may be regarded as a weighted mean value of the form $(\lambda\rho v/2d)_{\text{av}}\, v$.

For small pressure drops, the approximations used do not lead to significant errors. For large pressure drops, which are often encountered in the flow of gases with large discharges and small pipeline diameters, the errors can be substantially reduced by applying, section by section, the flow equations given below and by using the conditions of continuity of pressure and discharge in passing from one section to another.

According to the results obtained in [2], for the conditions specified above we may write the equations

$$\frac{\partial^2 H}{\partial x^2} = \frac{1}{k}\frac{\partial H}{\partial t}, \qquad \frac{\partial^2 Q}{\partial x^2} = \frac{1}{k}\frac{\partial Q}{\partial t}, \tag{2}$$

where the constant $k = \pi^2 d^3 g/2\lambda\alpha Q_{\text{av}}$; $Q_{\text{av}}$ is the mean weight discharge; the function

$$H(p) = \int_{p_0}^{p} \rho\, dp,$$

defined in the interval $p_0 < p < p_k$, is expressed, under the assumption of an isothermal process, by the relation (3)

$$H(p) = \frac{\beta}{\alpha}\left(e^{\alpha p} - e^{\alpha p_0}\right); \tag{3}$$

$\beta$ and $\alpha$ have the values

$$\alpha = \frac{\log(p_k/p_0)}{p_k - p_0}, \qquad \beta = \frac{(p_k + p_0)\log(p_k/p_0)}{2gRTZ\left(e^{\alpha p_k} - e^{\alpha p_0}\right)}, \tag{4}$$

$p_0$ and $p_k$ are the limits of pressure variation, respectively at the initial and terminal points of the pipeline; $T$ is the mean temperature and $Z$ the factor of deviation from the Boyle-Mariotte law.

In the case of laminar flow, the coefficient $k$, obviously, does not depend on the discharge and is equal to $k = d^2/32\,\mu\alpha$ ($\mu$ is the viscosity of the gas). For turbulent flow in the region for which the quadratic law is valid, the coefficient $\lambda$ depends only on roughness and does not depend on the Reynolds number.

Below we shall consider the case of a pipeline fed from a gas field, when the pressure at the initial point, after an interruption of inflow to the terminal point (we assume that the valve closes instantaneously), remains constant. As the initial condition we shall assume that the flow before the moment of closure was steady. Thus, we assume that at the initial point the gas flows at the maximum discharge corresponding to the difference between the pressure at this point and the pressure at the terminal point of the pipeline; the value of this difference corresponds to the discharge at the terminal point of the pipeline at the moment of closure and tends to zero after the pressure at this point becomes equal to the pressure at the initial point. It is evident that we are considering a limiting case, since in reality the boundary conditions at the initial point may vary from constant pressure to a discharge through this section equal to zero. For the case when, at the initial point of the pipeline, the pressure varies with time (after closure of the valve at the terminal point), the question is resolved by applying the principle of superposition, since equations (2) are linear.

Taking these considerations into account, we reduce the problem to the solution of a parabolic equation with the following boundary and initial conditions:

for $t > 0$

$$\frac{\partial H}{\partial x} = 0; \quad x = 0; \qquad H(l,t) = H_k, \quad x = l;$$

for $t = 0$

$$H(x) = \frac{H_k - H_0}{l}\,x + H_0. \tag{5}$$

Bearing in mind that as $t$ tends to infinity the pressure must be finite, and introducing the dimensionless quantities $\bar{H} = \dfrac{H - H_0}{H_k - H_0}$, $\bar{x} = \dfrac{x}{l}$, $\bar{t} = \dfrac{kt}{l^2}$, the solution of equation (2) may be written as a combination of known particular solutions in the form

$$\bar{H}(\bar{t},\bar{x}) = a_0 + a_1\bar{x} + \sum_{n=0}^{\infty}(a_n \cos n\alpha\bar{x} + b_n \sin n\alpha\bar{x})e^{-n^2\alpha^2\bar{t}}, \tag{6}$$

where the coefficients $a_0 = 1$, $a_1 = 0$, $b_n = 0$, and $a_n$ are determined by expanding the function $f(\bar{x}) = \bar{x} - 1$ in a Fourier series. After standard transformations, which are not given here, we obtain

$$a_{2m+1} = -8/\pi^2(2m+1)^2.$$

Taking relation (3) into account, one may write

$$\varphi(t) = \frac{e^{\alpha p} - e^{\alpha p_0}}{e^{\alpha p_k} - e^{\alpha p_0}} = 1 - \frac{8}{\pi^2} \sum_{m=0}^{\infty} \frac{\cos \frac{1}{2}\pi(2m+1)x/l}{(2m+1)^2} e^{-(2m+1)^2 \frac{\pi^2}{4} \frac{kt}{l^2}} \tag{7}$$

or, after substituting the pressure difference $(p_k - p_0)$ as a function of the weight flow rate before closure, assuming $Q_{\mathrm{av}} = Q$, we have

$$Q^2 = \frac{g^2 d^5 \pi^2}{8\lambda} \frac{\partial H}{\partial x} = \frac{g^2 d^5 \pi^2 \beta}{8\lambda\alpha} \left(e^{\alpha p_k} - e^{\lambda p_0}\right), \tag{8}$$

and for the pressure rise at the end point of the pipeline $x = 0$ the equation is obtained

$$e^{\alpha p_k} - e^{\alpha p} = \frac{64 Q^2 \lambda l \alpha}{\pi^4 d^5 g^2 \beta} \sum_{m=0}^{\infty} \frac{e^{-(2m+1)^2 \pi^2 kt/4l^2}}{(2m+1)^2}. \tag{9}$$

It is easy to see that, if the relation

$$\sum_{m=0}^{\infty} \frac{\cos \frac{1}{2}\pi(2m+1)x/l}{(2m+1)^2} = \frac{\pi^2}{8}\left(1 - \frac{x}{l}\right) \quad \text{for } t = 0,$$

is taken into account, the initial condition will be satisfied by equation (7).

Since, for numerical calculations in some practical cases, it is sufficient with good approximation to retain only the first term, equation (9), in semilogarithmic coordinates, represents a straight line with slope tangent $i = \frac{\pi^2}{4} \frac{k}{l^2}$, cutting off on the axis $\log(e^{\alpha p_k} - e^{\alpha p_0})$ the segment $A = \frac{64 Q^2 \lambda l \alpha}{\pi^4 d^5 g^2 \beta}$. Consequently, by recording graphically, with the aid of a sensitive differential manometer, the change of pressure with time at the end point after closing the valve, we can determine, by means of the relations given above, the actual values of the coefficients $\lambda$ and $\alpha$ or $\beta$.

Compared with the calculation method in which the equation of steady flow is used, the proposed method has the advantage that the value obtained for the hydrodynamic coefficient of friction applies to the entire length of the pipeline,

Fig. 1

Figure 1: Fig. 1

except for the zone of different roughness on its last section. The influence of this zone on the pressure rise with time, causing a deviation of the relation in semilogarithmic coordinates from a li-

linear, it can be excluded when processing the measurement results, using the section of the curve that becomes rectilinear some time after the inflow at the terminal cross-section of the pipeline has ceased.

By comparing the values found by means of methods of pressure recovery and steady-state flow, one can obtain valuable indications concerning the resistance to flow along the entire length of the pipeline relative to its last section.

**Fig. 1**

Figure 1 shows the increase in pressure with time for a trunk pipeline with the following characteristics (4): $d = 0.5$ m; $Q = 30$ kg/sec; $\lambda = 0.012$, $p_k = 50$ ata, $p_0 = 25$ ata, and $l = 300$ km; consequently, the stabilization time, i.e., the time required for the pressures to equalize, is approximately 7 hours.

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