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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

Theory of Elasticity

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Lamb’ s Problem for an Internal Source

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In connection with the study of wave processes arising in soils during underground explosions, it is of interest to consider the following problem of the dynamic theory of elasticity.

Let there be given, in a homogeneous isotropic elastic medium with Lamé parameters λ and μ and density ρ, bounded by a free surface, on some sphere of radius R_0 with center at the point O (Fig. 1), a source signal. It is usually customary to describe the signal by one of the functions: either $\dot{u}_R(t)$—the velocity of the particles along the radius from the point O, or $\sigma_R(t)$—the normal stress, and to assume that for $t < 0$ the medium is at rest, while the signal is switched on at the instant $t = 0$.

Fig. 1. Geometrical picture of waves in the vertical plane passing through the epicenter of the source and the point of observation. P_{inc}—longitudinal wave from the source; P_{refl}—wave reflected from the free surface; S—transverse wave arising upon reflection of P_{refl}; h—depth of the source; r—cylindrical coordinate.

The formal construction of the solution of the corresponding nonstationary boundary-value problem for the equations of the theory of elasticity at present presents no difficulty ([1,2]). Of main interest is the study of oscillograms of ground motion at interior points and on the free boundary of the elastic half-space ([3]) at distances comparable with the quantity V_pT—the “wavelength” (V_p is the velocity of propagation of longitudinal waves, T is some characteristic time of the signal).

In this note the displacement field $\mathbf{u}(r, z, t)$ on the daylight surface of the half-space is investigated, having components u_r, u_z; the signal is described by the function $\dot{u}_R(t) = u_0 f(t)\varepsilon(t)$, where $\varepsilon(t)$ is the unit function; $\varepsilon(t) = 0$ for $t < 0$; $\varepsilon(t) = 1$ for $t \geq 0$; $f(t)$ is a continuous function; $f(0) = 0$; u_0 is a constant of the dimension of velocity.

In this case the solution of the problem can be written in the form

$$\mathbf{u}(r, z, t) = \int_0^t f_1(t-\tau)\, \mathbf{u}_\varepsilon(r, z, \tau)\, d\tau, \tag{1}$$

where $\mathbf{u}_\varepsilon(r, z, \tau)$ is the solution of the posed problem for a concentrated signal varying in time according to the law $\varepsilon(t)$;

$$f_1(\tau) = f(\tau) - \int_0^\tau f(\nu) \exp[-(\tau-\nu)]\, d\nu; \qquad t = \frac{t^* V_p}{R_0}, \quad \tau = \frac{\tau^* V_p}{R_0};$$

t^* and τ^* are measured from the beginning of the process.

Using the methods proposed in (^{2}), $u_\varepsilon(r, z, \tau)$ can be represented in the form of the following expressions:

$$u_\varepsilon = u_{0\varepsilon} + u_{R\varepsilon} + u_{\lambda\varepsilon},$$

where each term admits a representation of the form

$$u_{i\varepsilon} = \frac{u_0 R_0}{V_s} \frac{1}{t} U_i(\xi, \eta, \gamma),$$

where $i = 0, R, \lambda$; U_i are dimensionless functions of the variables $\xi = r/V_p t^*$, $\eta = h/V_p t^*$.

The specific expressions for the components u_r have the form:

$$U_{r0} = \frac{\gamma}{1-\gamma^2} \frac{\xi}{(\xi^2+\eta^2)^{3/2}}, \qquad \gamma = \frac{V_s}{V_p}; \qquad a = \frac{1}{V_p},$$

$$U_{rR} = \frac{4\sqrt{1-\vartheta^2}}{\vartheta \xi d_1 \sqrt{\rho}} \left(\eta \sqrt{1-\gamma^2\vartheta^2} \sin\frac{\varphi}{2} + a v_0 \cos\frac{\varphi}{2} \right); \tag{2}$$

$$\rho = \sqrt{\left[\xi^2 + \eta^2(1-\gamma^2\vartheta^2) - v^2 a^2\right]^2 + 4 v_0^2 a^2 \xi^2 (1-\gamma^2\vartheta^2)};$$

$$\varphi = \operatorname{Arc\,tg} \frac{2 v_0 a \eta \sqrt{1-\gamma^2\vartheta^2}}{a^2 v_0^2 t^2 - \xi^2 - \eta^2(1-\gamma^2\vartheta^2)} + \frac{\pi}{2}; \qquad v_0 = \vartheta V_s;$$

ϑ is the root of the equation

$$(2-\vartheta^2)^2 = 4\sqrt{1-\vartheta^2}\sqrt{1-\gamma^2\vartheta^2};$$

$$d_1 = 4\left(2 - \vartheta^2 - \gamma^2\frac{\sqrt{1-\vartheta^2}}{\sqrt{1-\gamma^2\vartheta^2}} - \frac{\sqrt{1-\gamma^2\vartheta^2}}{\sqrt{1-\vartheta^2}}\right);$$

$$U_{r\lambda} = -\frac{4}{\pi\xi}\int_1^{1/\gamma}\frac{(2-\lambda^2)^2\sqrt{\lambda^2-1}}{(2-\lambda^2)^4 + 16(\lambda^2-1)(1-\gamma^2\lambda^2)}\times$$

$$\times\left(\eta\sqrt{1-\gamma^2\lambda^2}\sin\frac{\varphi_1}{2} + \gamma\lambda\cos\frac{\varphi_1}{2}\right)\frac{d\lambda}{\sqrt{\rho_1}};$$

ρ_1 and φ_1 are obtained from ρ and φ by replacing ϑ by λ. For the component u_z:

$$U_{z0} = -\frac{\gamma}{1-\gamma^2}\frac{\eta}{(\xi^2+\eta^2)^{3/2}}, \qquad U_{zR} = \frac{2(2-\vartheta^2)}{\vartheta d_1\sqrt{\rho}}\sin\frac{\varphi}{2},$$

$$U_{z\lambda} = -\frac{8}{\pi}\int_1^{1/\gamma}\frac{(2-\lambda^2)\sqrt{\lambda^2-1}\sqrt{1-\gamma^2\lambda^2}}{(2-\lambda^2)^4 + 16(\lambda^2-1)(1-\gamma^2\lambda^2)}\sin\frac{\varphi_1}{2}\frac{d\lambda}{\sqrt{\rho_1}}.$$

These formulas make it possible to carry out exact investigations of the displacement field on the surface of an elastic medium. To obtain the result it is necessary, in addition to numerical integration in (2) and (3), to perform integration according to (1) with a function that takes into account the real change in particle velocity with time, specified on a certain sphere of radius R_0. As R_0 one usually takes the distance from the center of the charge to the instrument, upon whose record the free surface of the medium has not yet had time to exert any substantial influence.

In Fig. 2 theoretical oscillograms are presented, constructed for the parameters indicated there; the form of the function $f(t)$ and the trajectory followed by a particle during the passage of the wave process are also indicated.

Two features of the displacement field on the surface $z = 0$ should be noted: 1) in comparison with motion in an unbounded medium, described by the signal, motion on the free surface is more complex: additional peaks and troughs appear —and it is more prolonged; 2) if the motion of a particle in an unbounded medium occurs along the radius from the point O, then the motion of the surface is characterized by a polarization that is different at the beginning and at the end of the process.

If we turn to Fig. 1, in which the wave fronts in the half-space at some instant of time are indicated schematically (this construction was carried out with the aid of a qualitative analysis of the constructed solution; see, for example, (2)), then the complex motion of the surface shown in Fig. 2b can be interpreted in the following way. The beginning of the motion of a surface point—upward and

Fig. 2

Figure 2: Fig. 2

forward from the epicenter—corresponds to motion in the longitudinal wave P. This motion takes place along an ellipse-like orbit with clockwise rotation. After a transition segment with motion of the particle downward and then upward, the particle begins to move along an ellipse-like orbit with counterclockwise rotation.

Fig. 2. a—records of the vertical (u_z) and horizontal (u_r) displacements of a point of the daylight surface as a function of the dimensionless time $t = t'V_p/R_0$, for the conventional vertical scale $k = u_0 R_0 / V_s$, and the variation of the signal $f(t)$ in time on an arbitrary vertical scale; $1.5h = R_0$; $r = 4R_0$; $\gamma = 1/\sqrt{3}$; $V_p =$ 4500 m/sec.; b—trajectory of motion of a surface particle for the records u_r and u_z shown in Fig. 2a.

Comparing the geometrical picture of the waves (Fig. 1) and the features of the motion noted in Fig. 2b leads to the conclusion that the transition segment with downward-upward motion may be identified with motion in the transverse waves S_j, whose envelope is the transverse wave S, while the final segment, with counterclockwise motion of the particle, may be identified with motion in the surface wave R (Rayleigh wave).

In concluding the analysis, it should be noted that the separation of the complex train of oscillations into individual waves is conditional and has the following meaning: the selected portions of the oscillograms and of the trajectory are identified with individual waves on the basis of kinematic features (propagation velocity, arrival time of the waves) and dynamic features (the character of the particle motion, relative amplitudes and frequencies, the law of attenuation of the maximum amplitudes), according to the predominant manifestation of the properties of a given wave on these portions. Generally speaking, one cannot deny the manifestation of longitudinal waves in the S portions, or the manifestation of transverse waves S in the R portion; however, this manifestation of additional disturbances is weaker than that of the principal ones, whose indices have been assigned to the portion.

The results of the analysis performed emphasize the necessity of additional and detailed quantitative investigations of the displacement field at relatively small distances from the explosion epicenter, and may find application in this case in estimating safe distances.

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Note: Figure translations are in progress. See original paper for figures.