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Abstract

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CYBERNETICS AND THE THEORY OF CONTROL

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QUALITATIVE ANALYSIS OF ELECTRICAL CIRCUITS WITH FINITE CONDUCTANCES BY MEANS OF THE THEORY OF CUMULATIVE NETWORKS

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1. One of the problems of nonlinear electrical engineering, brought forward by the development of the modern theory of contact circuits, is the mathematical analysis of circuits consisting of contacts and elements of finite conductance, i.e., conductance that cannot be set equal to 0 or ∞. In this case one may be concerned either with a complete quantitative analysis ([1–3]), or with a simplified qualitative analysis, when it is necessary only to ascertain the presence of zero, finite, or infinite conductance between the nodes of the circuit ([4]); sometimes in qualitative analysis attempts are also made to take account of various orders of finite conductance ([5,6]).

In the present paper new matrix methods are proposed for the qualitative analysis of circuits made up of contacts and elements of finite conductance. These methods are obtained by applying to the problem under consideration the apparatus, developed by the author, of the general combinatorial theory of cumulative networks ([7,8]), and serve as yet another illustration of the ability of this theory to embrace a broad range of problems in control and communications engineering. In their essence, these methods are a generalization of matrix methods for the analysis of purely contact circuits ([9–11]).

The proposed methods are suitable for the analysis of both two-terminal and multi-terminal circuits, both series-parallel and bridge circuits.

2. If on the set $\{0,\ {}^1/_2,\ 1\}$ one defines the operations $x + y = \max(x, y)$ and $xy = \min(x, y)$, then a distributive structure with zero and unit is obtained (cf. ([4,12–14])). This structure forms part of the three-valued logic of J. Łukasiewicz ([12,13]) and therefore may be called the **three-element algebra of Łukasiewicz**.

Theorem 1. *The three-element algebra of Łukasiewicz is an absorptive algebra in the sense of* ([7,8]).

Let now a circuit C consist of contacts and elements of finite conductance, and let f_{ij} be the conductance only of the branch directly joining node i with node

j, while F_{ij} is the total conductance of the entire circuit C between nodes i and j. We agree, instead of the words "the conductance is infinite," to say "the conductance is equal to 1," and instead of the words "the conductance is finite (but different from zero)," to say "the conductance is equal to $^1/_2$." Then the problem of qualitative analysis of the circuit C is reduced to the problem of computing the matrix $\|F_{ij}\|$ from the given matrix $\|f_{ij}\|$ in the three-element algebra of Łukasiewicz.

Lemma 1. $f_{ii} = F_{ii} = 1$.

Lemma 2. *The conductance of each path in C from node i to node j is equal in the algebra of Łukasiewicz to the product of the conductances of the branches entering into this path.*

Indeed, what is meant is the minimum of the conductances of the links of the path (i.e., the minimum of the estimates $0,\ ^1/_2,\ 1$).

Lemma 3. *The conductance F_{ij} is equal in the algebra of Łukasiewicz to the sum of the conductances of all paths from node i to node j.*

In fact, what is involved is the maximum conductance among all the conductances of these paths.

3. Let us apply the theory of cumulative networks.

Theorem 2. *Let $M = \{1, 2, \dots, p\}$ be the set of nodes of the circuit C. Then the system*

$$\hat{C} = \left\{ M,\ \|f_{ij}\|,\ \left\{ \sum_{r=2}^{n+1} \rho_r(M) \right\} \right\}$$

is a regular strong absorbent with final connectivity $\|f^*_{ij}\| = \|F_{ij}\|$.

Indeed, by Theorem 1 the three-element algebra of Łukasiewicz is an absorbent, and consequently the system $\hat{C}$ is an absorbent. It is strong by its expansion ([7,8]), and regular by Lemma 1. Therefore Theorem 3 of ([8]) guarantees the stability of the network $\hat{C}$ and the existence in it of final connectivity $\|f^*_{ij}\| = \|f^{(p-1)}_{ij}\|$. But $\|f^{(p-1)}_{ij}\| = \|F_{ij}\|$ by Lemmas 2 and 3.

Corollary 1. $\|F_{ij}\| = \|f_{ij}\|^\mu = \|f_{ij}\|^{p-1}$, where μ is the final number.

Indeed, see Theorem 6 of ([8]).

Corollary 2. $F_{kl} = |f_{ij\lambda lk}|kl$, where $|\ \ |_{kl}$ is the symbol of a quasiminor in the sense of ([11]).

Indeed, see Theorem 7 of ([8]).

Corollary 3. $F_{kl} = |f_{ij\lambda lk}|$, where $|\ \ |$ is the symbol of a signless minor in the sense of ([15]).

Indeed, see Theorem 8 of ([8]).

Thus, from the theory of cumulative networks we have simply and naturally derived three matrix methods for analyzing circuits made of contacts and elements of finite conductance, namely: 1) a method consisting in raising the matrix $\|f_{ij}\|$ to successive powers (Corollary 1); 2) a method consisting in computing quasiminors of the matrix $\|f_{ij}\|$ (Corollary 2); 3) a method consisting in computing signless minors of the matrix $\|f_{ij}\|$ (Corollary 3).

4. Our formulas for computing $|F_{ij}|$ from $|f_{ij}|$ apply directly to the case in which all conductances f_{ij} are the constants $0, \frac{1}{2}, 1$, i.e., when the contacts are assigned a fixed open or closed position. However, nothing prevents us from taking, as the elements of the matrix $\|f_{ij}\|$, variable conductances of contacts in the form of Boolean functions and multiplying them by $\frac{1}{2}$ or adding $\frac{1}{2}$ to them. Only in this case we shall be dealing not directly with the three-element algebra of Łukasiewicz, but with an algebra of functions induced by it, whose range of values is contained in this three-element algebra of Łukasiewicz.

Moreover, our methods of analysis remain valid even for circuits with contacts of multi-position switches, where instead of Boolean functions one must take one-place predicates ([16,17]).

5. The proposed methods could also be applied to the analysis of orders of conductance in the direction outlined in works ([5,6]). In this case, instead of the three-valued algebra of Łukasiewicz, one need only take the n-element algebra of Łukasiewicz, defined on the set

$$\left\{0, \frac{1}{n-1}, \frac{2}{n-1}, \dots, \frac{n-2}{n-1}, 1\right\}$$

with the same operations $x + y = \max(x, y)$,

$$xy = \min(x, y)$$

(see ([12,14])). Here the numbers $\frac{1}{n-1}, \frac{2}{n-1}, \dots, \frac{n-2}{n-1}$ must estimate the various orders of finite conductance. In all other respects the analysis procedure remains the same.

However, such an extension, in the author’ s opinion, will acquire significance only when adequate methods are developed for sorting elements of finite conductance by orders of conductance, since such sorting must precede the analysis. In contrast, the simple qualitative analysis discussed in §§ 2 and 3 has immediate practical significance even now.

6. Thanks to the isomorphism of the phenomena, Theorem 2 and the methods of analysis following from it can be applied in another domain—in the

general theory of processes, which underlies the theory of multicontact relay circuits[18] and may be briefly called **processology**.

Let us take, instead of an electrical circuit C, a diagram of processes in an arbitrary system S, so that the nodes of C will correspond to the states of the system S, the contacts of C to necessary (determinate) transitions between these states, and the elements of finite conductance to possible (probable, but not necessary) transitions between states. Then $\|f_{ij}\|$ will give a picture of the immediate connections between states, a picture of the elementary stages of processes in S, while $\|F_{ij}\|$ will give a picture of the complete connections between states, the general picture of the processes in S. Just as the matrix methods of analysis of purely contact circuits are applicable to the analysis of processes proceeding according to the principle "yes—no" or "possible—impossible" [19], so the matrix methods of analysis of circuits made up of contacts and elements of finite conductance are applicable to the analysis of processes proceeding according to the principle "impossible—possible—necessary."

Łukasiewicz' s three-valued algebra appears here as a genuine **modal logic**. However, the qualitative analysis of electrical circuits discussed in the preceding paragraphs can also, in a certain sense, be interpreted as tracing the logical connection of events (cf. [20]).

The extension of our methods considered in § 5 acquires, in the field of processology, a quite real meaning if systems are studied in which transitions between states have n modalities: from "impossibility" 0 through various levels of the "possible" to "necessity" 1. Łukasiewicz' s n-element algebra will here be an n-valued modal logic.

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Note: Figure translations are in progress. See original paper for figures.