ON ENERGY SPECTRA IN THE PASSAGE OF BETA RADIATION THROUGH MATTER

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Abstract

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The energy distribution of β-radiation at various points of an irradiated medium is of great importance in studies using radioactive isotopes, especially in a number of biophysical problems connected with determining the biological effectiveness of radiation at the cellular level.

We consider a plane source of β-radiation with a specified distribution of electrons over energies $f(\eta)$, placed in an isotropic homogeneous medium. In the passage of β-radiation through matter, both electron scattering and their energy losses for ionization and excitation of the atoms of the medium are taken into account. The only assumption is that these losses are regarded as continuous. The electron transport equation ([1]) for our case may be written in the form

$$-\frac{\partial I(\xi,\vartheta,\eta)}{\partial\eta}+\mu\frac{\partial I(\xi,\vartheta,\eta)}{\partial\xi}=$$

$$=2\pi\int_{-1}^{+1}Ns_0\sigma(\eta,\theta)\left[I(\xi,\vartheta',\eta)-I(\xi,\vartheta,\eta)\right]d\mu''+(4\pi)^{-1}\delta(\xi)f(\eta). \qquad (1)$$

Here $I(\xi,\vartheta,\eta)$ is the electron distribution function; N is the number of scattering atoms in 1 g of substance; $\sigma(\eta,\theta)$ is the cross section for scattering of electrons by 1 atom of the medium into a unit solid angle; s_0 is the greatest range in the given medium of an electron with maximum energy $E_{\max}$; ξ is the position of the electron in units of s_0; η is the real path traversed by the electron from the source to the point ξ, in units of s_0. Taking into account the relation between the path traversed and the electron energy, one may regard η as an energy parameter. The dependence $\eta=\varphi(\varepsilon)$ is shown in Fig. 1, b. ε is the electron energy in units of $E_{\max}$; $\mu=\cos\vartheta$; ϑ determines the direction of motion of the electron; $\mu''=\cos\theta$; $\theta=|\vartheta'-\vartheta|$.

The analysis carried out of the experimental data makes it possible to choose $f(\eta)$ in the form:

$$f(\eta)=C\eta^a(1-\eta)^2, \qquad (2)$$

where the constant a is different for different isotopes, with $0 < a \leqslant 1$.

To isolate the angular dependence, we represent the solution of (1) in the form of an expansion in Legendre polynomials

$$I(\xi, \eta, \vartheta) = (4\pi)^{-1} \sum_{l=0}^{\infty} (2l+1) P_l(\mu) I_l(\xi, \eta). \tag{3}$$

If we restrict ourselves to isotopes with $E_{\max} \lesssim 1$ MeV, then the functions $I_l(\xi, \eta)$ are determined by the equation

$$\frac{\partial I_l(\xi,\eta)}{\partial \eta} - \frac{d_l}{\eta} I_l(\xi,\eta) = \frac{l+1}{2l+1} \frac{\partial I_{l+1}(\xi,\eta)}{\partial \xi} + \frac{l}{2l+1} \frac{\partial I_{l-1}(\xi,\eta)}{\partial \xi} - \delta(\xi)\delta_{l,0} f(\eta); \tag{4}$$

$\delta_{l,0}$ is the Kronecker symbol. Using as $\sigma(\eta, \theta)$ the usual Rutherford scattering cross section, in which, to take account of the screening effect, $\operatorname{cosec}^4 \theta/2$ is replaced by $0.25(1+2\chi-\cos\theta)^{-2}$, and the factor $(z+1)z^{-1}$, which takes inelastic collisions into account, is added, we obtain

$$d_l = \frac{(z+1)}{4B} \frac{d}{d\chi} \left[Q_l(1+2\chi) - Q_0(1+2\chi) \right], \tag{5}$$

where χ is the screening constant [2], and $Q_l(t)$ are Legendre functions of the second kind [3].

Let us introduce the double moments of the function $I_l(\xi, \eta)$:

$$I^{(\alpha)}_{l,n,m} = \int_{-1}^{+1} d\xi\, \xi^n (1-|\xi|)^{1-\alpha} \int_0^1 d\eta\, \eta^m I_l(\xi,\eta). \tag{6}$$

The recurrence relation for determining them is obtained directly from (4):

$$(d_l + m + 1)(2l+1) I^{(\alpha)}_{l,n,m} = n(l+1) I_{l+1,n-1,m+1} + nl I_{l-1,n-1,m+1}, \tag{7}$$

$$l = 1, 2, \ldots; \quad n = 1, 2, \ldots; \quad m = 1, 2, \ldots;$$

$$I_{0,0,m} = (m+1)^{-1} v^{(\alpha)}_{0,m+1}, \tag{7'}$$

where we have denoted

$$v^{(\alpha)}_{k,m} = \int_0^1 \eta^{\alpha+m} (1-\eta)^{2-k}, \qquad k = 0, 1, 2. \tag{8}$$

We seek the solution of (4) in the form of an expansion in certain functions $\Omega_i(\xi,\eta)$, close to the expected asymptotic behavior of the solution of equation (4). The coefficients of this expansion are determined with the aid of the moments (6)

$$I_0(\xi,\eta)=\sum_{i=0}^{N}p_i\Omega_i(\xi,\eta), \tag{9}$$

with the conditions

$$I_0(0,\eta)=f(\eta); \qquad I_0(1,\eta)=I_0(\xi,0)=I_0(\xi,\eta_{\max})=0, \tag{10}$$

where

$$\eta_{\max}=1-|\xi|. \tag{11}$$

We choose the functions $\Omega_i(\xi,\eta)$ as follows:

$$\Omega_i(\xi,\eta)=C\eta^{\alpha}\left(1-\eta-\frac{|\xi|}{q_i}\right)^2\left(\frac{|\xi|}{q_i}\right)^i\left(1-\frac{|\xi|}{q_i}\right)^{-2}u(q_i-|\xi|)\times$$

$$\times\exp\left\{-\frac{A|\xi|}{q_i}\left(1-\frac{|\xi|}{q_i}\right)^{-i}\right\}; \tag{12}$$

$$u(x)=\begin{cases}1, & x>0,\\ 0, & x<0.\end{cases} \tag{13}$$

To use the entire interval of variation of ξ, we set $q_0=1$. Three of the conditions (10) are fulfilled by the choice of the functions. The first gives $p_0=1$.

Restricting ourselves to $N=1$, taking (6), (9), and (12) into account, we obtain a system of equations for determining the constants q_1 and p_1:

$$I_{0,\,2n,\,0}=v_{0,\,0}\{\overline{\Phi}_{2n}+p_1q_1^{2n+1}\overline{\Phi}_{2n+1}\}, \qquad n=0,\ 1; \tag{14}$$

where

$$\Phi=\omega_n-2\omega_{n+1}+\omega_{n+2}; \tag{15}$$

$$\overline{\omega}_n=2\,\frac{n!}{A^{n+1}}\left[1-e^{-A}\sum_{i=0}^{n}\frac{A^n}{n!}\right]; \tag{16}$$

Fig. 1. Energy spectra of C14: 1–for $\xi = 0.0000$, 2–0.06935, 3–0.13870, 4–0.20806, 5–0.34677, 6–relation between the energy parameters η and ϵ

Figure 1: Fig. 1. Energy spectra of C14: 1–for $\xi = 0.0000$, 2–0.06935, 3–0.13870, 4–0.20806, 5–0.34677, 6–relation between the energy parameters η and ϵ

$$\omega_n = 2\sum_{k=0}^{n}(-1)^k\binom{n}{k}\frac{1}{(k+1)!}\times$$

$$\times\left[(-1)^k A^{k+1}\,\mathrm{Ei}(-A)e^A + \sum_{j=0}^{k+1}(-1)^{j+1}(k+i-j)!\,A^{j-1}\right]. \qquad (17)$$

The change in the number of particles $N(\xi)$ with distance is determined from (9) by integration over η:

$$\ln\frac{N(\xi)}{N(0)} = -A|\xi| + (1+\alpha)\ln(1-|\xi|). \qquad (18)$$

Here we have neglected the quantity Δ

$$\Delta = p_1\left(1-\frac{|\xi|}{q_i}\right)^{2+\alpha}(1-|\xi|)^{-(1+\alpha)}u(q_i-|\xi|)\exp\left\{-\frac{A|\xi|-q_i|\xi|+|\xi|^2}{q_i-|\xi|}\right\}. \qquad (19)$$

Equation (18) can be used to determine the constant A. As an example, we considered a C^{14} source placed in polystyrene. For the initial $f(\eta)$ we took the experimentally measured spectrum; α proved to be equal to 0.13, the constant $A = 7.6$, and the calculated values of p_1 and q_1 were, respectively, 20.626 and 0.04979. In Fig. 1, 1-5 show the theoretical (lines) and experimental (points) values of the energy spectra at various penetration depths ξ. Some discrepancy between theory and experiment for intermediate values of ε is explained by the insufficiently accurate choice of the analytical form of the initial β-radiation spectrum, which we sought to specify by an expression simple and universal for different isotopes.

Fig. 1. Energy spectra of C^{14}: 1–for $\xi = 0.0000$, 2–0.06935, 3–0.13870, 4–0.20806, 5–0.34677, 6–relation between the energy parameters η and ε.

The theory considered was also successfully applied to the isotopes S^{35} and Ca^{45} in polystyrene.

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Note: Figure translations are in progress. See original paper for figures.