ON A MODEL OF A CAVITATING LIQUID

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Abstract

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ON A MODEL OF A CAVITATING LIQUID

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It should be noted that there is as yet no sufficiently satisfactory mathematical model describing the flow of a liquid when cavitation bubbles are present within it. The work of Ackeret ([1]), who assumed that the bubbles present in the liquid are compressed adiabatically and that the pressure in the water coincides with the pressure inside the bubbles, is one of the first attempts to construct such a model.* A certain modification of these ideas was obtained in ([2]), where a diffusion model of a cavitating liquid was constructed. However, in the indicated theories the dynamics of individual bubbles was not taken into account.

In the present note a model of a medium is proposed which, in a certain approximation, may be regarded as a cavitating liquid, and in which the expansion and compression of bubbles obey the ordinary equations of hydrodynamics. The onset of cavitation depends essentially on the presence in the liquid of foreign particles– "cavitation nuclei" (of sizes 10^{-3}–10^{-5} cm), around which cavitation bubbles form and grow ([3–6]).

For the proposed model of the medium it is assumed:

1) The "cavitation nuclei," which are spheres of identical radius R_0, are distributed uniformly throughout the entire volume of an incompressible liquid of density ρ_0, so that per unit mass of liquid there are n such particles.
2) Cavitation bubbles arise in the liquid as soon as the pressure becomes less than a certain value p_{cr}.
3) The pressure inside the cavitation bubbles during their expansion and compression remains constant and equal to p_{cr}.
4) The radial motion of all bubbles occurs according to the same law as the motion of an individual spherical cavity with constant internal pressure in an infinite volume of incompressible liquid. Then for each bubble the equation is valid

$$R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 = \frac{p_{\text{cr}} - p}{\rho_0}, \tag{1}$$

where R is the radius of the bubble at time t; p is the pressure in the liquid; by d/dt is meant the substantial derivative.

5) The mixture of liquid and vapor is taken to be a homogeneous medium with density equal to the mean density of the mixture. Neglecting the volume of evaporated liquid in comparison with the volume of the vapor formed, the mean the density can be calculated by the formula

$$\rho = \frac{\rho_0}{1 + b\left(R^3 - R_0^3\right)}, \qquad b = \frac{4}{3}\pi n \rho_0. \tag{2}$$

Equations (1) and (2) establish a relation between pressure and density

$$p = f\left(\rho, \frac{d\rho}{dt}, \frac{d^2\rho}{dt^2}\right). \tag{3}$$

The equation of continuity, the equation of conservation of momentum, and equation (3) form a closed system of equations. For the given medium one can determine the free energy and entropy in the following way:

$$F\left(R, \frac{dR}{dt}, T\right) = \frac{3b}{2} R^3 \left(\frac{dR}{dt}\right)^3 - \frac{b}{\rho_0} p_{\text{cr}} R^3 + \varphi(T), \qquad S = -\frac{\partial F}{\partial T} \tag{4}$$

In continuous motion no energy losses occur, since

$$dQ' = -dF - S\,dT + \frac{p}{\rho^2}\,d\rho = 0, \tag{5}$$

where dQ' is the "uncompensated" heat.

It is easy to construct a generalization of the proposed model allowing for surface-tension forces, viscosity, and variability of the pressure inside the bubbles.

Thus, the motion of the liquid is described by the equations of motion of an incompressible liquid until, somewhere, the pressure becomes less than the value p_{cr}. From this moment on, the motion of the liquid, which will no longer be incompressible, is governed by the indicated system of equations. This system will remain valid until the value R again becomes equal to R_0. If at this moment the radial velocity of the bubbles is nonzero, then energy is lost due to inelastic impact.

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* Ackeret proceeded from the assumption that the mixture of liquid and bubbles may be regarded as a certain hypothetical homogeneous compressible medium.

Let us now consider the steady one-dimensional flow of a liquid through a tube with a local contraction. If in some section S_0, located upstream of the minimum section S_{min}, the pressure falls to the value p_{cr}, then below this section a cavitation region begins. The equations of motion in this region take the form

$$\rho U \frac{dU}{dx} = -\frac{dp}{dx},$$

$$\rho u S = Q = \rho_0 u_0 S_0,$$

$$\rho = \frac{\rho_0}{1 + b\left(R^3 - R_0^3\right)},$$

$$u^2 \left[R \frac{d^2 R}{dx^2} + \frac{3}{2} \left(\frac{dR}{dx} \right)^2 \right] + uR \frac{du}{dx} \frac{dR}{dx} = \frac{p_{\text{cr}} - p}{\rho_0}, \tag{6}$$

where u is the flow velocity and Q is the discharge. As initial conditions in the section S_0 we have

$$R = R_0, \qquad \rho = \rho_0, \qquad u = u_0, \qquad p = p_{\text{cr}}. \tag{7}$$

The system of equations (6) with initial conditions (7) was integrated numerically for the case when the tube shape is given by the equation

$$S(x) = k_0 x^2 + k_1. \tag{8}$$

Depending on the values of the parameters k_0, k_1 and the initial data, two types of outflow are possible:

1) The bubbles grow, starting from radius R_0, reach a certain maximum value, and then compression of the bubbles begins. We assume that

cavitation ceases when the value of the radius R reaches the value R_0, and thereafter an incompressible fluid again flows.

2) The bubbles exhibit a tendency toward continuous growth. Obviously, this case corresponds to supersonic flow of a compressible fluid through a nozzle, and therefore, for an arbitrarily prescribed value of the pressure at the outlet of the tube p_2, it is impossible to construct a continuous solution. It becomes necessary to introduce a shock, coinciding with the boundary of the cavitation region. At the shock we require fulfillment of the conditions of conservation of mass and momentum and assume that the density of the fluid after the shock coincides with the initial density ρ_0. The coordinate of the shock x_c is related to the pressure in the final section p_2 by the relation

$$p_{\mathrm{cr}} - p_2 + \frac{\rho_0 u_0^2}{2}\left[1 - \left(\frac{S_0}{S_2}\right)^2\right] = b\rho_0 u_0^2 S_0^2 \int_{x_0}^{x_c} \frac{R^3 - R_0^3}{S^3}\frac{dS}{dx}\,dx. \tag{9}$$

This dependence is obtained by integrating the first equation of system (6), taking into account the conditions at the shock. The expression on the right-hand side of formula (9) determines the energy losses arising during cavitation.

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References

[1] J. Ackeret, Forsch. auf d. Gebiete des Ingenieurwesens, Ausgabe A, **1**, 63 (1930).
[2] L. A. Épshtein, Tr. TsAGI, No. 584 (1946).
[3] R. T. Knapp, Pap. Am. Soc. Mech. Eng., No. A-80 (1957).
[4] M. Strasberg, Cavitation in Hydrodinamics, London, 1956, 6—1—6—13.
[5] M. Strasberg, J. Acoust. Soc. Am., **31**, No. 2, 162 (1959).
[6] K. S. Iyengar, E. G. Richardson, Brit. J. Appl. Phys., **9**, No. 4, 154 (1958).

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