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Abstract

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PHYSICS

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ON DISPERSION RELATIONS IN NONLOCAL FIELD THEORY

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1. Usually the existence of dispersion relations is associated with the causality of the theory, as is the case in ordinary local field theory. The question of dispersion relations in nonlocal field theory was discussed by a number of authors ([1,2]), who proceeded from the condition $[j(x),\, j(y)] = 0$ for $(x-y)^2 > l_0$. However, this condition is not a condition of macrocausality and cannot be used to obtain dispersion relations. The errors in papers ([2,3]) consist in using, in order to obtain dispersion relations, an expression of the scattering matrix element through $T(j(x),\, j(y))$, which does not hold in nonlocal field theory, since in deriving this expression the usual local commutation relations were essentially used ([3]).

Below we investigate the question of obtaining dispersion relations using the macrocausality condition obtained in ([4]).

2. In ([4]) the following estimate was obtained

$$\int_a^\infty |T^R|\, z_0^n dz_0 \leq f \sum_{m=0}^{n} \frac{a^{n-2m-1}(2\sqrt{2})^{m+1} B^2_{\frac{m+1}{2}}\, n!}{(u-m)!}, \tag{1}$$

where T^R is the scattering matrix element; a is the width of the wave packet, multiplied by $C = \min_r |\operatorname{Im} C_r|$; C_r is a pole of the form factor; $B_s = \int |V^{(s)}(x)|\, dx$; $V(x)$ is the wave packet. The derivative is taken with respect to time. For noninteger s we shall take the nearest larger integer.

Let us estimate the growth of the quantities B_s, assuming here that the wave packets are such that B_s grow most slowly. We have

$$B_{2s} = 4 \sum_i V^{(2s-1)}(a_i^{(2s)}), \qquad B_{2s+1} = 4 \sum_i V^{(2s)}(a_i^{(2s+1)}) + 2V^{(2s)}(0);$$

$a_i^{(s)}$ are the roots of the s-th derivative. It is easy to see that the inequality holds

$$\left|V^{(s)}(a_i^{(s+1)})\right| > \frac{V^{(s-1)}(a_i^{(s)})}{\Delta a_i^{(s)}}; \qquad \Delta a_i^{(s)} = a_i^{(s)} - a_{i-1}^{(s)}.$$

Consequently,

$$B_s > 2\sum_{i=0}^{s} \frac{1}{\Delta a_i^{(s)} \Delta a_{i-1}^{(s-1)} \dots \Delta a_1^{(k)} \Delta a_1^{(k-1)} \dots \Delta a_1^{(1)}}.$$

We have taken $B_0 = 1$.

Since $\sum_i \Delta a_i^{(s)} = 1$, the B_s will be minimal for $\Delta a_1^{(s)} = \Delta a_2^{(s)} = \dots = \Delta a_s^{(s)} \dfrac{1}{s+1}$. Then $B_s > 2(s+1)(s+1)!$, or $B_s \approx 2\alpha^{s+1}(s+1)(s+1)!$, where α is not much greater than 1.

Now we shall assume that, at least for large z_0, the estimate

$$|T^R| \leq ke^{-\lambda|z_0|}$$

holds.

From the requirement that (1) be satisfiable for arbitrary a and n, we obtain

$$\lambda \geq (1/a\sqrt{2})^{1/2} \sim 1,$$

i.e., the "elementary length" $l_0 \approx 1/c$.

3. We shall now consider the scattering of a scalar neutral particle of mass m by a scalar charged particle, analogously to paper (4). Choosing the reference frame in which the momentum of the scatterer $\mathbf{p} = 0$, for forward scattering we obtain

$$T^{R'}(E) = \frac{1}{(2\pi)^{3/2}} \int \exp\left[iEz_0 - ez\sqrt{E^2 - m^2}\right] T^R(z)\, dz$$

$$= \frac{4\pi}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} dz_0\, e^{iEz_0} \int_0^{\infty} |z|\, d|z|\, \frac{\sin|z|\sqrt{E^2 - m^2}}{\sqrt{E^2 - m^2}}\, T^R(z), \qquad (2)$$

Introduce the quantity

$$T_l^R(z) = \begin{cases} T^R(z), & z_0 < l, \\ 0, & z_0 \geq l. \end{cases}$$

Then

$$T^R(E) = \frac{4\pi}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} e^{iEz_0}\, dz_0 \int_0^{\infty} \frac{|z| \sin|z|\sqrt{E^2 - m^2}}{\sqrt{E^2 - m^2}}\, T_l^R(z)\, d|z| + \omega_l^R(E), \qquad (3)$$

where

$$\omega_l^R(E) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{+\infty} dz\, \exp\left[-ez\sqrt{E^2 - m^2}\right] \int_l^{\infty} e^{iEz_0} T^R(z)\, dz_0.$$

From (3) it is easy to establish that the quantity

$$e^{-iEl}(T^R - \omega_l^R)$$

can be analytically continued into the lower half-plane by the method used in (5). One can likewise introduce the quantity

$$e^{iEl}(T^A - \omega_l^A)$$

and construct a function analytic in the upper half-plane, with the exception of cuts along the real axis $(-\infty, -m)$, $(m, +\infty)$. The dispersion relations are obtained without particular difficulty. However, they will contain ω_l^R and ω_l^A, which can only be estimated. To estimate them we introduce the quantity

$$T_1^R = z_1^2 z_2^2 z_3^2 T^R.$$

According to equality (9) of paper (4),

$$T_1^R(z) \simeq \int e^{ikz} \frac{\partial^6}{\partial k_1^2\, \partial k_2^2\, \partial k_3^2} \left[(k^2 - m^2)\, \tilde{A}(k)\, \tilde{V}_X(k)\, \tilde{V}_{X+Z}(k)\right] d\bar{k}.$$

After differentiation, one obtains a sum of expressions containing derivatives of the Fourier transforms of the wave packets, which will lead to consideration, instead of wave packets, of the quantities $x_j^{(s)}\tilde{V}(x)$; but these quantities belong to the same class as $V(x)$, and the character of the estimates for $T_1^R(z)$ remains the same as for $T^R(z)$.

The estimate for ω_l^R will have the form

$$|\omega_l^R| \leq \frac{4\pi}{3} \int_l^{\infty} |T^R|\, dz_0 + 12\pi \int_l^{\infty} |T_l^R|\, dz_0.$$

It follows from the estimate for T^R that $\left|\omega_l^R\right| \ll \text{const} \cdot l_0 e^{-l/l_0}$. We see that, for sufficiently small l, the difference from the usual dispersion relations will be negligible. In conclusion, we note that we have operated with wave packets of finite extent, although in reality such packets cannot be realized, since their Fourier transforms must contain frequencies of one sign. However, real packets will differ very little from those used by us, and estimates can be made of the influence of their "tails."

The results obtained indicate that, even if a modification of the dispersion relations in nonlocal field theory proves impossible, nevertheless for a small "elementary length" the usual "local" dispersion relations are almost satisfied.

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Note: Figure translations are in progress. See original paper for figures.