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Abstract

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INTERACTION OF ROTATIONAL DISCONTINUITIES

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The interaction of shock waves and rarefaction waves in gas dynamics was considered in papers (^{1,2}). The interaction of magnetohydrodynamic shock waves and rarefaction waves with one another, and also with contact and rotational discontinuities, was investigated in papers (^{3,4}). In the present paper we consider the interaction of rotational discontinuities (A), which is of special interest. At the instant of collision a discontinuity arises on which the relations of the conservation laws are not satisfied; therefore it decomposes into certain combinations of waves. The aim of the work is to determine the corresponding combinations of waves.

From the relations on rotational discontinuities (^{5}) it follows that on the discontinuity arising in the collision

$$p_0 = p_0', \qquad |\mathbf{H}_{\tau 0}| = |\mathbf{H}_{\tau 0}'|, \qquad u_0 = u_0' = 0;$$

$\Delta\vec{v}_\tau$ satisfies the relations written below.

The notation used coincides with that adopted in paper (^{6}). Let us recall that, in the decay of a discontinuity, the index 0 ($0'$) denotes the parameters of the undisturbed medium lying at the initial moment of time to the left (right) of the discontinuity; the index 1 ($1'$) denotes the parameters of the medium behind the first wave, moving relative to the contact discontinuity to the left (right), etc. In the collision of rotational discontinuities, by the index 1 we shall denote the parameters of the undisturbed medium, and by the index 0 ($0'$)—the parameters of the medium behind the A-discontinuity moving to the right (left).

Let us first consider a somewhat more general problem—the decay of a discontinuity for which $p_0 = p_0'$, $|\mathbf{H}_{\tau 0}| = |\mathbf{H}_{\tau 0}'|$, $u_0 = u_0' = 0$, but $\mathbf{H}_{\tau 0}$ and $\mathbf{H}_{\tau 0}'$ are arbitrary. The jump of the velocity at the discontinuity $\Delta\vec{v}_\tau$ may also be arbitrary. Let the angle between $\mathbf{H}_{\tau 0}$ and $\mathbf{H}_{\tau 0}'$ be equal to α.

The solution of this problem (^{6}) consists in constructing diagrams in the space $\Delta u\,\Delta v\,\Delta w$, with the aid of which, knowing Δu, Δv, Δw, one can indicate the combination of waves and discontinuities that is the solution of the problem. From § 12 of paper (^{6}) it follows that, in the case under consideration, the diagram in the space $\Delta u\,\Delta v\,\Delta w$ can be obtained from the diagram for the case of the decay of a discontinuity for which $p_0 = p_0'$, $\mathbf{H}_{\tau 0} = \mathbf{H}_{\tau 0}'$, by a corresponding translation of the latter relative to the origin of coordinates. As shown in paper (^{4}), the diagram for the case $p_0 = p_0'$, $\mathbf{H}_{\tau 0} = \mathbf{H}_{\tau 0}'$ coincides with the diagram shown in Fig. 1 of paper (^{7}), except that different combinations correspond to the lines and regions. Thus, to the lines and regions to which the combinations V^+, P^-V^+, V^-R^+, etc. (in our notation S^+, R^-S^+, S^-R^+, etc.) correspond, in the case of the discontinuity under consideration there correspond, respectively, the combinations S^+KS^+, $S^+R^-KR^-S^+$, $R^+S^-KS^-R^+$, i.e., on both sides there travel the waves indicated in Fig. 1 of paper (^{7}).

Let the spatial discontinuity under consideration decay into two A-waves traveling to both sides. From the relations on rotational disconti-

from (5) it follows that

$$\vec{v}_{\tau 1} = \vec{v}_{\tau 0} + (\mathbf{h}_1 - \mathbf{h}_0)V_0, \qquad \vec{v}_{\tau 1}' = \vec{v}_{\tau 0}' - (\mathbf{h}_1' - \mathbf{h}_0')V_0'.$$

The equation of the line in the space $\Delta u\ \Delta v\ \Delta w$ that corresponds to the decay into two A-waves is (6)

$$[\Delta\vec{v}_\tau - (\mathbf{h}_0 + \mathbf{h}_0')V_0]^2 = 4h_0^2V_0^2.$$

It is clear that, in the diagram in the space $\Delta u\ \Delta v\ \Delta w$, this equation is that of the circle σ, lying in the plane $\Delta u = 0$, of radius $2h_0V_0$ with center at the point

$$\Delta v^0 = h_0V_0(1+\cos\alpha), \qquad \Delta w^0 = h_0V_0\sin\alpha.$$

The direction $\mathbf{H}_{\tau 0}$ has been chosen as the y-axis; the x-axis is perpendicular to the front of the discontinuity. The circle intersects the Δv-axis at the points

$$\Delta v_{1,2} = h_0V_0(1+\cos\alpha) \pm h_0V_0\sqrt{4-\sin^2\alpha}.$$

For $\alpha = 0$,

$$\Delta v^0 = 2h_0V_0, \qquad \Delta w^0 = 0, \qquad \Delta v_1 = 4h_0V_0, \qquad \Delta v_2 = 0.$$

The circle under consideration becomes the corresponding line in Fig. 1 from (7)—a circle of the same radius, lying in the plane $\Delta u = 0$, with center on the Δv-axis. The angle $\alpha = 0$ corresponds to a discontinuity on which $\mathbf{H}_{\tau 0} = \mathbf{H}_{\tau 0}'$. When α varies from 0 to ±180°, the radius of the circle σ does not change, while its center moves along a curve that is also a circle of radius h_0V_0, with center lying on the Δv-axis at a distance h_0V_0 from the origin, and shown in Fig. 1 by the continuous thin line.

Fig. 1

Figure 1: Fig. 1

Fig. 1

For $\alpha = 90^\circ$,

$$\Delta v^0 = \Delta w^0 = h_0 V_0, \qquad \Delta v_{1,2} = h_0 V_0 (1 \pm \sqrt{3}).$$

The angle $\alpha = 90^\circ$ corresponds to a discontinuity on which $\mathbf{H}_{\tau 0} \perp \mathbf{H}'_{\tau 0}$. The position of the circle σ for $\alpha = 90^\circ$ is shown in Fig. 1 by the continuous heavy line.

For $\alpha = 180^\circ$,

$$\Delta v^0 = \Delta w^0 = 0, \qquad \Delta v_{1,2} = \pm 2 h_0 V_0.$$

The angle $\alpha = 180^\circ$ corresponds to a discontinuity on which $\mathbf{H}_{\tau 0}$ and $\mathbf{H}'_{\tau 0}$ are antiparallel. According to the results of § 11 of [6], the qualitative picture in the plane Δu Δv for $\alpha = 180^\circ$ remains the same as in the case $\alpha = 0$, except that instead of combinations without rotational discontinuities and with two rotational discontinuities one must write combinations with one rotational discontinuity, traveling to the right or to the left. It was indicated that if $H_{y0} > 0$, $H'_{y0} < 0$, then combinations with a rotational discontinuity traveling to the right are located below the separation line, and with a rotational discontinuity traveling to the left–above it. Indeed, if $H_{y0} > 0$, $H'_{y0} < 0$ ($\alpha = 180^\circ$), the point $\Delta v_1 = 2h_0 V_0$, $\Delta w_1 = 0$ corresponds to one A-discontinuity traveling to the left, while the point $\Delta v_1 = -2h_0 V_0$, $\Delta w_1 = 0$ corresponds to one A-discontinuity traveling to the right.

The circle σ under consideration in the case $\mathbf{H}_{\tau 0} = \mathbf{H}'_{\tau 0}$ is the section, by the plane $\Delta u = 0$, of the surface formed by rotating the lines that correspond to the combinations S^+KS^+, R^+KR^+. Above, the displacement of this circle in the case when $\mathbf{H}_{\tau 0}$ is not parallel to $\mathbf{H}'_{\tau 0}$ was considered in detail. Exactly

Similarly, one can consider the displacement of any section of the surface, and hence of the whole surface.

As is readily seen from Fig. 1 of [7], outside the circle σ there is a region to which the decay of the discontinuity into the combination $S^+AR^-KR^-AS^+$ corresponds; to the points of the circle itself there corresponds decay of the discontinuity into the combination AA; inside the circle there is a region to which decay into the combination $R^+AS^-KS^-AR^+$ corresponds.

Let us consider the collision of A-discontinuities, the discontinuities being allowed to have arbitrary intensity. From the relations on rotational discontinuities [5] it follows that

$$\vec{v}_{\tau 0} = -(\mathbf{h}_0 - \mathbf{h}_1)V_0, \qquad \vec{v}\,'_{\tau 0} = (\mathbf{h}'_0 - \mathbf{h}'_1)V'_0, \qquad |\mathbf{h}_0| = |\mathbf{h}'_0|.$$

Hence

$$[\Delta\vec{v}_\tau + (\mathbf{h}_0 + \mathbf{h}'_0)V_0]^2 = 4h_0^2V_0^2.$$

Let the angle between $\mathbf{H}_{\tau 0}$ and $\mathbf{H}'_{\tau 0}$ be equal to γ. This angle determines the difference in intensities of the colliding discontinuities.

From the last formula it is clear that, when two rotational discontinuities collide, the end of the vector $\Delta\vec{v}_\tau$ lies on the circle δ in the plane $\Delta u = 0$, of radius $2h_0V_0$, with center at the point

$$\Delta v^{00} = -h_0V_0(1 + \cos\gamma), \qquad \Delta w^{00} = -h_0V_0\sin\gamma.$$

The direction $\mathbf{H}_{\tau 0}$ has been chosen as the y-axis. The circle passes through the points

$$\Delta v_{1,2} = -h_0V_0(1 + \cos\gamma) \pm h_0V_0\sqrt{4 - \sin^2\gamma}, \qquad \Delta w_{1,2} = 0.$$

As the values of the angle γ vary from 0 to ±180°, the center of the circle moves in the plane $\Delta v\,\Delta w$ along a curve which is also a circle of radius h_0V_0, with center lying on the Δv axis at a distance $-h_0V_0$ from the origin, and shown in Fig. 1 by a thin dashed line,

$$\begin{aligned}
&\Delta v^{00} = -2h_0V_0, \quad \Delta w^{00} = 0, \quad \Delta v_1 = 0, \quad \Delta v_2 = -4h_0V_0 && \text{for } \gamma = 0\ (\mathbf{H}_{\tau 0} \parallel \mathbf{H}'_{\tau 0});\\
&\Delta v^{00} = \Delta w^{00} = -h_0V_0, \quad \Delta v_{1,2} = -h_0V_0(1 \pm \sqrt{3}) && \text{for } \gamma = 90^\circ\ (\mathbf{H}_{\tau 0} \perp \mathbf{H}'_{\tau 0});\\
&\Delta v^{00} = \Delta w^{00} = 0, \quad \Delta v_{1,2} = \pm 2h_0V_0 && \text{for } \gamma = 180^\circ.
\end{aligned}$$

The position of the circle δ corresponding to the value $\gamma = 90^\circ$ is shown in Fig. 1 by a heavy dashed line. As is seen from Fig. 1, this circle may be tangent to the circle σ ($\gamma = \alpha = 0$), may intersect it ($-180^\circ < \gamma = \alpha < 180^\circ$), and may coincide with it entirely ($\gamma = \alpha = \pm 180^\circ$). To the points of the circle δ there corresponds the magnitude of the jump $\Delta\vec{v}$ arising in the collision of A-discontinuities. Consequently, the points corresponding to different $\Delta\vec{v}$ may lie inside the circle σ, on the circle itself, and outside it. Accordingly, in the collision of A-discontinuities there arises a discontinuity which may decay into the following combinations: 1) $S^+AR^-R^-AS^+$, if $\gamma = 0$; 2) $S^+AR^-R^-AS^+$, $R^+AS^-S^-AR^+$, AA, if $-180^\circ < \gamma < 180^\circ$, the type of combination depending on the parameters h_0, V_0, γ; 3) AA, if $\gamma = \pm 180^\circ$.

Fig. 2

Figure 2: Fig. 2

We note that a contact discontinuity does not arise in the decay of the discontinuity formed by the collision of A-discontinuities. The coordinates of the intersection points of the circles are found in an elementary way.

Let us consider the collision of discontinuities of equal intensity with $\gamma = 0$. In this case $\vec{v}_{\tau 0}$ and $\vec{v}'_{\tau 0}$ are antiparallel. From the consideration given above it follows that the discontinuity formed as a result of the collision must decay into the combination $S^{+}AR^{-}R^{-}AS^{+}$.

From symmetry it is clear that the problem of the collision of two rotational discontinuities of equal intensity ($\gamma = 0$) is equivalent to the problem of the collisio-

of a rotational discontinuity with an ideally conducting wall. Consequently, after collision the combination of waves $S + AR^{-}$ will be reflected from the wall.

If one considers the collision process in a coordinate system in which the medium behind the rotational discontinuity is at rest, then the problem under consideration is equivalent to the problem of a piston moving in a medium at rest with velocity $\vec{v}_{\mathrm{p}}$, equal in absolute value to the velocity of the medium behind the rotational discontinuity, but directed to the opposite side; moreover, the velocity component u_{p} normal to the plane of the piston is equal to zero.

Generally speaking, when a piston moves in a conducting medium [7] with a velocity lying in the plane of the piston, various combinations of waves may propagate ahead of the piston (see Fig. 2). To the points of the inner circumference there corresponds a single rotational discontinuity traveling ahead of the piston; to the points of the line B, and also to the points lying beyond this line, there corresponds the combination of waves $S + AR^{-}$ traveling ahead of the piston, in which the wave R^{-} has maximum intensity, so that a vacuum is formed ahead of the piston; to the points of the Δv axis there correspond combinations of the same kind as for the regions, but the intensity of the rotational discontinuity is equal either to zero (below the point O) or to 180° (above the point O). The occurrence, in the collision of an A-discontinuity with a wall, of only one combination of waves ($S + AR^{-}$) is due to the fact that behind the A-discontinuity v is always positive. (Here, for definiteness, it is assumed that the A-discontinuity rotating the tangential component of the magnetic field through an angle β moves to the left; the direction of the y-axis coincides with the direction of $\mathbf{H}_{\tau}$ behind the incoming A-discontinuity.)

Fig. 2

Let us show this. By the index 0 (1) we shall here denote the parameters of the medium behind (ahead of) the A-wave. Then

$$\vec{v}_{\tau 0} = (\mathbf{h}_0 - \mathbf{h}_1) V_0.$$

Hence

$$v = h_0 V_0 (1 - \cos\beta) > 0, \qquad w = \pm h_0 V_0 \sin\beta.$$

Consequently, $v_{\mathrm{p}} < 0$. It is seen from Fig. 2 that to points $v_{\mathrm{p}} < 0$ there corresponds the combination $S + AR^-$. In the collision with a wall of a plane rotational discontinuity ($\beta = 180^\circ$), the combination $S + R^-$ ($w_{\mathrm{p}} = 0$) will be reflected from the wall.

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Note: Figure translations are in progress. See original paper for figures.