PHYSICS

B. T. VAVILOV

1961

SovietRxiv

Abstract

Full Text

PHYSICS

B. T. VAVILOV

ON THE QUESTION OF THE ANGULAR DISTRIBUTION IN MULTIPLE PROCESSES

(Presented by Academician V. A. Leontovich, 12 X 1960)

For the analysis of the angular distribution of mesons arising in the collision of two nucleons with very high energy, we apply the method developed in ([1]) for constructing a system of coupled equations for the matrix $V^{ij,nm,kl}$, describing the transition from the initial state $ink\rangle$ with i bosons, n fermions, and k antifermions to the final state $jml\rangle$.

We shall proceed from a scalar meson–nucleon interaction. Let us consider a model in which antifermions are absent both in the final and in the virtual states, i.e., mesons interact with one another only as a consequence of scattering on a real nucleon. The analysis of such a model may be useful for clarifying whether the deviation of the angular distribution from an isotropic one is a consequence only of the meson-meson interaction, or whether it exists even when the latter is neglected. Let us write the system of coupled equations for the matrix $V^{0n,22,00}$ graphically in the form

(1)

(2)

where the rectangle with m incoming and m outgoing nucleon ends and n outgoing meson ends is the graphical representation of the operator $V^{0n,mm,00}$. The system (1)–(2) includes an infinite number of equations. Its exact solution does not correspond to any finite approximation in perturbation theory and is essentially equivalent to summing an infinite number of irreducible “ladder” diagrams, the number of internal meson lines in which varies from 1 to infinity.

In what follows we shall assume $n \gg 1$, which, obviously, corresponds to the case of very high energies. In addition, as in ([1]), we shall use C-number fermion distribution functions (the approximation

Bloch–Nordsieck) [(2)*]

$$S^c(p) = i(2\pi)^{-4}(ap - m)^{-1}, \tag{3}$$

where p is the 4-momentum, m is the nucleon mass, and a is a 4-vector satisfying the condition $a^2 = 1$.

We seek the solution of (1)–(2) in the form

$$V^{0n,22,00}[\sigma,\sigma_0] = \int_{\sigma_0}^{\sigma} d^4\xi \sum_{\text{imp}} \bar{u}(p_1)\bar{u}(p_2)u(q_1)u(q_2) \prod_{\alpha=1}^{n} \varphi^{(+)}(k_\alpha) Q_n \times \exp\left\{ i\xi \left(p_1 + p_2 - q_1 - q_2 + \sum_{\alpha=1}^{n} k_\alpha \right) \right\}, \tag{4}$$

$$V^{0n,11,00}[\sigma,\sigma_0] = \int_{\sigma_0}^{\sigma} d^4\xi \sum_{\text{imp}} \bar{u}(p)u(q) \prod_{\alpha=1}^{n} \varphi^{(+)}(k_\alpha) B_n \exp\left\{ i\xi \left(p - q + \sum_{\alpha=1}^{n} k_\alpha \right) \right\}. \tag{5}$$

We assume that forward-backward symmetry holds, and, consequently, for $n \gg 1$ in the center-of-mass system (C-system)

$$\sum_{\alpha=1}^{n} k_\alpha = 0,$$

where k_α is the momentum of the α-th meson. Then, in the C-system, Q_n is a function of the initial nucleon energy E, the inelasticity coefficient γ, and the meson momenta k_i $(i = 1, 2, \dots, n)$. Substituting (4) and (5) into (1) and (2), we obtain for Q_n and B_n the equations

$$\begin{aligned} P(k_1, k_2, \dots, k_n) \Big\{ Q_n(E,\gamma; k_1, \dots, k_n) - (2\pi)^4 \frac{g}{2i} \left[S^c(p_1 + k_n) + S^c(p_2 + k_n) \right] \\ \times Q_{n-1}(E,\gamma; k_1, \dots, k_{n-1}) - (2\pi)^8 g^2 Q_n(E,\gamma; k_1, \dots, k_n) \\ \times \int d^4k \, P(p_1,p_2) \left[S^c(p_1 - k) D^c(k) S^c(p_2 + k) \right] \Big\} \\ = \frac{(2\pi)^4}{4i} g n P(q_1,q_2) P(p_1,p_2) P(k_1,\dots,k_n) \\ \times \left[D^c(q_2 - p_2) B_{n+1}(q_1, p_1, k_1, \dots, k_n, q_2 - p_2) \right], \end{aligned} \tag{6}$$

$$\begin{aligned} B_n(q,p,k_1,\dots,k_n) = (2\pi)^4 \frac{g}{in!} P(k_1,\dots,k_n) \\ \times \left[S^c(p + k_n) B_{n-1}(q, p + k_n, k_1, \dots, k_{n-1}) \right], \end{aligned} \tag{7}$$

where $P(k_1, \dots, k_n)$ is the operator of summation over all possible $n!$ permutations of the indices $(1, 2, \dots, n)$; $E_{p_1} = E(1-\gamma)$, $\mathbf{p}_1 \simeq E(1-\gamma)\mathbf{e}$, and $\mathbf{e}$ is the unit vector determining the orientation of $\mathbf{p}_1$**. By virtue of the condition

$$\sum_{\alpha=1}^{n} k_\alpha = 0,$$

obviously, $E_{p_2} = E_{p_1}$, $\mathbf{p}_2 = -\mathbf{p}_1$.

The simultaneous solution of (6) and (7) leads to the following result:

$$Q_n = \frac{g^{n+2}}{2n!} f_n(g; E, \gamma) P(p_1, p_2) \times$$
$$\times \sum_{l=0}^{n} P\left(\frac{k_1 \dots k_l}{k_{l+1} \dots k_n}\right) \prod_{j=1}^{l} \left[a\left(p_1 + \sum_{\alpha=1}^{j} k_\alpha\right) - m \right]^{-1} \cdot \prod_{i=l+1}^{n} \left[a\left(p_2 + \sum_{\alpha=1}^{j} k_\alpha\right) - m \right]^{-1}, \tag{8}$$

* The system of units is chosen so that $\hbar = c = 1$.

** Here it is assumed that the energy of the nucleons in the final state remains ultrarelativistic, so that the rest masses may be neglected in comparison with the kinetic energy.

where $P\binom{k_1 \dots k_l}{k_{l+1} \dots k_n}$ is the operator of summation over all $\frac{n!}{l!(n-l)!}$ partitions of the set $(k_1 \dots k_n)$ into $(k_1 \dots k_l)$ and $(k_{l+1} \dots k_n)$, with simultaneous symmetrization within the latter, and $f_n(g; E, \gamma)$ satisfies the equation

$$\left\{ 1 - \frac{ig^2}{(2\pi)^4} \int d^4k \, P(p_1, p_2) [a(p_1 - k) - m]^{-1} (k^2 - \mu^2)^{-1} [a(p_2 + k) - m]^{-1} \right\} \times$$
$$\times f_n(g; E, \gamma) - f_{n-1}(g; E, \gamma) = (2\pi)^4 \frac{g}{i} n P(q_1, q_2) P(p_1, p_2) \times$$
$$\times [D^c(p_1 - q_1) - D^c(p_2 - q_2)]. \tag{9}$$

Since the momenta k_α do not appear in (9), the angular distribution of the mesons is completely described by the sum of products entering formula (8). It is easy to see that (8) does not possess spherical symmetry, i.e., the angular distribution is anisotropic.

Denoting the probability of meson emission symmetrically forward—backward by $W_n(0; \pi)$, and the probability of the distribution of meson momenta in the plane perpendicular to $\mathbf{p}_1$ and passing through the center of mass by $W_n(\pi/2)$, we obtain

$$\eta = \frac{W_n(\pi/2)}{W_n(0;\pi)} = \prod_{j=1}^{n}\left[1 - \frac{(\alpha_0^2 E^2 - m^2)^{1/2}\sum_{\alpha=1}^{j}|\mathbf{k}_\alpha|}{\alpha_0 E\sum_{\alpha=1}^{i}\omega_\alpha}\right]^2, \tag{10}$$

where $\alpha_0 = 1 - \gamma$.

Thus, even without taking into account the π—π interaction, the theory leads to an angular distribution in the c.m. system that differs from an isotropic one, with maxima corresponding to emission symmetrically forward and backward. The distribution becomes close to isotropic only for very slow mesons, when $|\mathbf{k}_\alpha| \to 0$.

Comparing the above considerations with the "shaking" theory[3], we come to the conclusion that the isotropy obtained in[3] is apparently a consequence of the assumption that there is no substantial interference between the interaction processes and that there is no recoil effect of the radiation on the source. The inclusion of antinucleons in the consideration greatly complicates the solution of the full system of equations for the matrix $V^{0n,\,22,\,00}$. One of the other ways of taking the π—π interaction into account consists in introducing an interaction term $\lambda\varphi^4$ into the Hamiltonian. Preliminary rough estimates have shown that, when this term is taken into account, the angular distribution changes somewhat, although it remains anisotropic.

I express my gratitude to V. I. Grigor' ev for useful discussions.

Moscow State University
named after M. V. Lomonosov

Received
10 X 1960

CITED LITERATURE

[1] B. T. Vavilov, V. I. Grigor' ev, *ZhETF*, **39**, 3 (9), 794 (1960).
[2] N. N. Bogolyubov, D. V. Shirkov, *Introduction to the Theory of Quantized Fields*, Moscow, 1957.
[3] H. W. Lewis, I. R. Oppenheimer, S. A. Wouthuysen, *Phys. Rev.*, **73**, 127 (1948).
[4] Proceedings of the International Conference on Cosmic Rays, Publishing House of the USSR Academy of Sciences, 1960.

Note: Figure translations are in progress. See original paper for figures.