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Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

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ON THE THEORY OF PULSE AUTOMATIC SYSTEMS WITH AMPLITUDE-PULSE MODULATION OF THE SECOND KIND

(Presented by Academician B. N. Petrov, 26 I 1961)

Amplitude-pulse automatic systems (a.p.s.) can be divided into two classes, depending on the kind of amplitude-pulse modulation.

In an a.p.s. I, the sequence of pulses consists of pulses of identical shape, whose heights are proportional to the values of the input (modulating) quantity at the discrete instants of time $t = nT$ (amplitude-pulse modulation of the first kind).

In an a.p.s. II, the output quantity of the pulse element is a sequence of pulses of duration γT, where $0 < \gamma \leq 1$, and T is the repetition period; moreover, the tops of these pulses vary according to the law of variation of the input (modulating) quantity of the pulse element (amplitude-pulse modulation of the second kind*).

The theory of a.p.s. I has been developed in sufficient detail ($^{1-3}$).

A.p.s. II may be regarded as continuous systems containing, in the error circuit, a switch that periodically closes the circuit (for a time γT) and opens it (for a time $(1-\gamma)T$) (Fig. 1). As a result, a.p.s. II are systems with abruptly varying parameters corresponding to the parameters of the open and closed continuous systems. Note that a.p.s. II for $\gamma = 1$ reduce to continuous systems, and for $\gamma \ll 1$ to a.p.s. I.

In the few works devoted to a.p.s. II ($^{3-6}$), a non-direct route was used to derive the exact equations of these systems.** First, on the basis of one or another form of the method of matching the solutions of closed and open continuous systems, a difference equation was formed that relates the values of the error (or of another quantity) at different discrete instants of time. Then the discrete Laplace transform (or, equivalently, the z-transform) was applied to these difference equations, and the transform corresponding to the values of the process at the discrete instants of time $t = nT$ was found.

To formulate the equations of a.p.s. II one can apply the theory of systems with periodically varying parameters (^{7}). However, this path leads to determinants of infinite order, whose computation is associated with great difficulties.

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Fig. 1

Figure 1: Fig. 1

In the present work a direct method is set forth for obtaining the equations of a.p.s. II with respect to the transforms of the discrete Laplace transform.

In contrast to a.p.s. I, the equations of a.p.s. II are not algebraic but integral equations of Fredholm type with a degenerate kernel. The solution of these equations determines the transform of the discrete Laplace transform, to which the results of the theory of a.p.s. I are directly applicable.

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* In a number of works, in particular in (8), the opposite definition of the kind of modulation is adopted.

** We do not touch here upon works in which approximate equations are obtained by replacing an a.p.s. II with an approximating a.p.s. I.

As shown in (8), the equation of the open-loop AIS II can be represented, with respect to images in the sense of the discrete Laplace transform (1), in the form

$$Z^*(q,\varepsilon) = \int_0^\gamma W_0^*(q,\varepsilon-\lambda)\,X^*(q,\lambda)\,d\lambda, \tag{1}$$

where

$$W_0^*(q,\varepsilon-\lambda) = \begin{cases} W^*(q,\varepsilon-\lambda), & \text{for } 0 \leqslant \lambda \leqslant \varepsilon, \\ e^{-q}W^*(q,1+\varepsilon-\lambda), & \text{for } \varepsilon \leqslant \lambda \leqslant 1. \end{cases} \tag{2}$$

The transfer function $W^*(q,\varepsilon)$ is equal to (1):

$$W^*(q,\varepsilon) = \sum_{\nu=1}^{l} c'_\nu \frac{e^q}{e^q - e^{q_\nu}}\, e^{q_\nu \varepsilon}, \tag{3}$$

where q_ν are the poles of the transfer function of the continuous part $W(q)$ (for simplicity they are assumed to be simple), and $c'_\nu = \lim_{q\to q_\nu}(q-q_\nu)W(q)$ are constants; $q = pT$ is a dimensionless parameter.

Fig. 1

For the closed-loop AIS II the closing condition is valid, which, with respect to images, is written in the form

$$X^*(q,\varepsilon) = F^*(q,\varepsilon) - Z^*(q,\varepsilon). \tag{4}$$

Eliminating $Z^*(q,\varepsilon)$ from (1) and (4), we obtain the equation of the closed-loop AIS II:

$$X^*(q,\varepsilon) = F^*(q,\varepsilon) - \int_0^\gamma W_0^*(q,\varepsilon-\lambda)\, X^*(q,\lambda)\, d\lambda. \tag{5}$$

Taking into account the different analytic representation of $W_0^*(q,\varepsilon-\lambda)$ for $\lambda < \varepsilon$ and $\lambda > \varepsilon$ (2), and noting that the difference of these analytic expressions is equal to the impulse response

$$w(\varepsilon-\lambda) = \sum_{\nu=1}^{l} c'_\nu e^{q_\nu(\varepsilon-\lambda)}, \tag{6}$$

we transform equation (5) to the form

$$X^*(q,\varepsilon) = F^*(q,\varepsilon) - \int_0^\varepsilon w(\varepsilon-\lambda)\, X^*(q,\lambda)\, d\lambda -$$

$$- \int_0^\gamma e^{-q} W^*(q, 1+\varepsilon-\lambda)\, X^*(q,\lambda)\, d\lambda. \tag{7}$$

This integral equation may be regarded as a combination of an integral equation of convolution type and a Fredholm equation for $0 \leqslant \varepsilon \leqslant \gamma$. Solving it first as an equation of convolution type, i.e., temporarily regarding the second integral as a known function, we reduce this equation to an ordinary Fredholm integral equation, which, after simple transfor-

expansions and substitution of the values $W^*(q,\varepsilon)$ and $w(\varepsilon-\lambda)$ will have the form:

$$X^*(q,\varepsilon) = F^*(q,\varepsilon) - \sum_{\mu=1}^{l} A_\mu e^{\overline{q}_\mu \varepsilon} \int_0^\varepsilon e^{-\overline{q}_\mu \lambda} F^*(q,\lambda)\, d\lambda -$$

$$- \sum_{\nu=1}^{l} \frac{c'_\nu e^{q_\nu}}{e^q - e^{q_\nu}} \sum_{\mu=1}^{l} \frac{A_\mu e^{\overline{q}_\mu \varepsilon}}{q_\nu - \overline{q}_\mu} \int_0^\gamma e^{-q_\nu \lambda} X^*(q,\lambda)\, d\lambda. \tag{8}$$

Here $\overline{q}_\mu$ are the poles of the transfer function of the closed continuous system

$$K_z(q) = \frac{W(q)}{1+W(q)},$$

$$A_\mu = \lim_{q\to\overline{q}_\mu} (q-\overline{q}_\mu) K_z(q)$$

are constant quantities.

The integral equation (8) has a degenerate kernel and can be solved by a known method (9). Denote

$$F_1^*(q,\varepsilon) = F^*(q,\varepsilon) - \sum_{\mu=1}^{l} A_\mu e^{\overline{q}_\mu \varepsilon} \int_0^{\varepsilon} e^{-\overline{q}_\mu \lambda} F^*(q,\lambda)\, d\lambda,$$

$$\Psi_\nu^*(q,\varepsilon) = \frac{c'_\nu e^{q_\nu \varepsilon}}{e^q - e^{q_\nu}} \sum_{\mu=1}^{l} \frac{A_\mu e^{\overline{q}_\mu \varepsilon}}{q_\nu - \overline{q}_\mu},$$

$$\xi_\nu = \int_0^{\gamma} e^{-q_\nu \lambda} X^*(q,\lambda)\, d\lambda. \tag{9}$$

Then equation (8) is written briefly in the form

$$X^*(q,\varepsilon) = F_1^*(q,\varepsilon) - \sum_{\nu=1}^{l} \Psi_\nu^*(q,\varepsilon)\xi_\nu. \tag{10}$$

Multiplying both sides of equation (10) by $e^{-q_s \varepsilon}$ and integrating with respect to ε over the limits from 0 to γ, we obtain a system of linear equations

$$\xi_s = B_s^*(q) - \sum_{\nu=1}^{l} E_{s,\nu}^*(q)\xi_\nu, \qquad s = 1, 2, \ldots, l, \tag{11}$$

where

$$B_s^*(q) = \int_0^{\gamma} e^{-q_s \varepsilon} F_1^*(q,\varepsilon)\, d\varepsilon, \tag{12}$$

$$E_{s,\nu}^*(q) = \int_0^{\gamma} e^{-q_s \varepsilon} \Psi_\nu^*(q,\varepsilon)\, d\varepsilon = \frac{c'_\nu e^{q_\nu}}{e^q - e^{q_\nu}} \sum_{\mu=1}^{l} \frac{A_\mu}{q_\nu - \overline{q}_\mu} \frac{e^{(\overline{q}_\mu - q_s)\gamma} - 1}{\overline{q}_\mu - q_s}.$$

The solution of the system of linear equations (11) with respect to ξ_k is equal to:

$$\xi_k = \sum_{s=1}^{l} \frac{\Delta_{ks}^*(q)}{\Delta^*(q)} B_s^*(q), \tag{13}$$

where

$$\Delta^*(q) = \begin{vmatrix} 1+E^*_{11}(q) & E^*_{12}(q) & \dots & E^*_{1l}(q) \\ E^*_{21}(q) & 1+E^*_{22}(q) & \dots & E^*_{2l}(q) \\ \dots & \dots & \dots & \dots \\ E^*_{l1}(q) & E^*_{l2}(q) & \dots & 1+E^*_{ll}(q) \end{vmatrix} \tag{14}$$

is the determinant of the system, and $\Delta^*_{ks}(q)$ are the algebraic cofactors corresponding to the elements with indices k and s. Substituting (13) into (10), we find the solu-

of the integral equation, determining the transform of the error $X^*(q,\varepsilon)$ in the sense of the discrete Laplace transform:

$$X^*(q,\varepsilon) = F^*_1(q,\varepsilon) - \sum_{\nu=1}^{l} \Psi^*_\nu(q,\varepsilon) \sum_{s=1}^{l} \frac{\Delta_{ks}(q)}{\Delta(q)} B^*_s(q). \tag{15}$$

This transform corresponds to the processes in a pulse automatic system II for $0 \leqslant \varepsilon \leqslant \gamma$. To determine $X^*(q,\varepsilon)$ for $\gamma \leqslant \varepsilon \leqslant 1$, under the integral sign in (5), instead of $W^*_0(q,\varepsilon-\lambda)$ one must substitute $W(q,\varepsilon-\lambda)$ from (2) and $X^*(q,\varepsilon)$ from (15). By the known results of the theory of pulse systems of class I, the process is found from the transforms obtained, or the properties of this process are determined. Thus, for example, the study of the stability of a pulse automatic system II is reduced to the study of the location of the zeros of determinant (14) in the complex q-plane, which is carried out on the basis of the stability criteria for pulse automatic systems I ([1]). Thus, the expression obtained for $X^*(q,\varepsilon)$ for a pulse automatic system II reduces the study of these systems to the study of the well-investigated pulse automatic systems I. Using the relation between the transform of the ordinary and the discrete Laplace transforms ([1]),

$$X(q) = \int_0^1 e^{-q\varepsilon} X^*(q,\varepsilon)\, d\varepsilon, \tag{16}$$

one can also find, from expression (15), the transform $X(q)$, to which, for example, the usual frequency methods for investigating continuous systems are applicable.

The approach described above extends to pulse automatic systems II with a delay in the continuous part and with a periodically varying repetition period.

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