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CYBERNETICS AND CONTROL THEORY

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1961

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Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

P. I. CHINAEV

ON ONE METHOD FOR DETERMINING TRANSFER MATRIX FUNCTIONS OF MULTILoop AUTOMATIC SYSTEMS

(Presented by Academician E. S. Kulebakin, 15 VII 1960)

The practice of developing complex automatic-control systems requires extensive use of methods for the analysis and synthesis of multiloop automatic systems. The initial data in this case are matrix differential equations, which mathematically reflect the physical processes occurring in the system and the relations between them (¹).

The method of transfer functions introduced by A. V. Mikhailov (²) is at present considered the principal method in composing the initial equations of motion of a single-loop system. However, for systems with many controlled coordinates, purely algebraic techniques are usually used, which complicates the calculations and makes the occurrence of errors in carrying them out more probable.

Below a method is proposed for determining the transfer matrix function for a complex multiloop automatic system.

Fig. 1. Structural diagram of a multiloop automatic system

Suppose there is a multiloop automatic system consisting of m loops, in each of which there are m elements with transfer functions $W_{iv}^k(p)$ ($i = 1, 2, \dots, m$; $v = 1, 2, \dots, m$). The structural diagram of the system is shown in Fig. 1. Between the loops, in the general case, there are direct and feedback connections. The law of connection and its sign are specified by the transfer function W_{ij}^c ($i \neq j$). For a direct connection the index d is used, and for feedback the index f . The case of a multiloop system with elements of the same type has been considered repeatedly (³).

Let us open the system loops along their main connection. Next, let us isolate from the entire system an m -terminal network ⁴ and construct its transfer matrix function. Suppose at first that there are direct positive connections between the loops (Fig. 2). We shall call such an m -terminal network an m -terminal

network of the first kind. Not all connections are shown in Fig. 2, but it should be assumed that from each point a there issue $m - 1$ connections and the same number enter each point b .

The transfer function of an m -terminal network of the first kind has the form

$$V_k = \begin{vmatrix} W_{11}^k & W_{21}^{\text{pc}} & \dots & W_{m1}^{\text{pc}} \\ W_{12}^{\text{pc}} & W_{22}^k & \dots & W_{m2}^{\text{pc}} \\ \cdot & \cdot & \cdot & \cdot \\ W_{1m}^{\text{pc}} & W_{2m}^{\text{pc}} & \dots & W_{mm}^k \end{vmatrix}. \quad (1)$$

Along the main diagonal are located the transfer functions of the loop elements. The remaining elements of the matrix are filled with the transfer functions of the connections, with their signs; moreover, above the main diagonal are placed the transfer functions of the lower (in the figure) connection elements, and below the diagonal—the transfer functions of the upper connection elements. In the absence of a connection, a zero is placed in the corresponding position of the matrix. If the connection remains direct but is negative, then the matrix elements are written with a minus sign. For example, the common case of a multiloop system with antisymmetric connections (Fig. 2 B) has a matrix whose symmetrically placed elements differ only in sign.

For the cascade connection of m -terminal networks,

$$X_{\text{out}}^{k-1} = X_{\text{in}}^k; \quad (2)$$

X_k is a column matrix.

Suppose there are s cascade-connected m -terminal networks of the first kind with transfer matrix functions V_1, V_2, \dots, V_s . What should be the equivalent transfer matrix of the new m -terminal network obtained by the cascade connection of s m -terminal networks?

Theorem. *For the cascade connection of s m -terminal networks of the first kind, the transfer matrix function of the equivalent m -terminal network is equal to the product of the s transfer matrix functions of the individual m -terminal networks of the first kind, taken in reverse order:*

$$W_I = V_s V_{s-1} \dots V_2 V_1. \quad (3)$$

Let us now consider an m -terminal network of the second kind (Fig. 3). We shall denote negative feedback by a minus sign before the transfer function ($-W^{\text{oc}}$). In this case

$$U_k X^k = X^{k-1},$$

Fig. 2. Structure of an m -terminal network of the 1st kind: A –symmetric couplings; B –nonsymmetric couplings

Figure 2: Fig. 2. Structure of an m -terminal network of the 1st kind: A –symmetric couplings; B –nonsymmetric couplings

Fig. 3. Structure of an m -terminal network of the 2nd kind

Figure 3: Fig. 3. Structure of an m -terminal network of the 2nd kind

where

$$U_k = \begin{pmatrix} \frac{1}{W_{11}^k} & W_{21}^{\text{oc}} & \cdots & W_{m1}^{\text{oc}} \\ W_{12}^{\text{oc}} & \frac{1}{W_{22}^k} & \cdots & W_{m2}^{\text{oc}} \\ \cdot & \cdot & \cdot & \cdot \\ W_{1m}^{\text{oc}} & W_{2m}^{\text{oc}} & \cdots & \frac{1}{W_{mm}^k} \end{pmatrix}. \quad (4)$$

If $|U_k| \neq 0$, then $X^k = U_k^{-1} X^{k-1}$.

The matrix transfer function of an m -terminal network of the 2nd kind is a matrix inverse to a matrix transfer function of the form (4). On the main diagonal of such a matrix are placed the inverse transfer functions of the principal loop elements, and the remaining elements are the transfer functions of the coupling elements. Above the main diagonal are placed the transfer functions of the lower elements in the drawing, and below the diagonal those of the upper ones; moreover, the transfer functions of the couplings are taken with the opposite sign: for negative feedbacks the transfer matrices W_{ij}^{oc} will everywhere have a plus sign. As in the preceding case, when couplings are absent, zeros are placed in lieu of those matrix elements which correspond to the broken couplings.

Fig. 2. Structure of an m -terminal network of the 1st kind: A –symmetric couplings; B –nonsymmetric couplings

Fig. 3. Structure of an m -terminal network of the 2nd kind

Theorem. *When two m -terminal networks of the 2nd kind are connected in cascade, the matrix transfer function of the equivalent m -terminal network is found as the product of the matrices inverse to the matrix (4), taken in reverse order*

$$W_{\text{II}} = U_s^{-1} U_{s-1}^{-1} \cdots U_2^{-1} U_1^{-1}. \quad (5)$$

Let us now consider the mixed case, when m -terminal networks of the 1st and 2nd kinds are connected in cascade (Fig. 4). The equation of each of them will

be written respectively in the form

$$\begin{aligned} X_1 &= V_1 X_0 & (X_0 = X_{\text{in}}), \\ U_2 X_2 &= X_1, \\ U_3 X_3 &= X_2, \\ &\dots \\ X_{s-1} &= V_{s-1} X_{s-2}, \\ X_s &= V_s X_{s-1} & (X_s = X_{\text{out}}), \end{aligned}$$

where V_k are matrices of the form (1), and U_k of the form (4). Eliminating the intermediate column matrices X_1, X_2, \dots, X_{s-1} , we obtain

$$W = V_s V_{s-1} \dots U_3^{-1} U_2^{-1} V_1. \quad (6)$$

A series connection of m -poles of different kinds is nothing other than an open-loop multiloop system. Therefore the following is valid:

Theorem. *The transfer matrix function of an open-loop multiloop system is equal to the product of the transfer matrix functions of the individual m -poles, taken in reverse order.*

Let us form the transfer matrix function of the closed-loop multiloop system. Since, for an open-loop system composed of s m -poles,

$$X_s = W X_0, \quad (7)$$

then for the closed-loop system

$$X_s = W(X_0 - X_s)$$

or

$$(E + W)X_s = W X_0; \quad (8)$$

E is the identity matrix.

Hence

$$X_s = \Phi X_0; \quad (9)$$

Φ is the transfer matrix function of the closed-loop system,

$$\Phi = (E + W)^{-1} W.$$

The equation of motion of a multiloop automatic system can be given in the form (8) or (9).

The method set forth for constructing the transfer matrix function can be applied to complex power systems and power units, to complex cybernetic systems, to production complexes, to computers, and to other automatic systems. Similar techniques are used in radio engineering and electrical engineering ⁽⁵⁾.

Fig. 4. Series connection of m -poles

If the system contains elements with delay (with distributed parameters) or discrete elements, it is sufficient to use, instead of ordinary transfer functions, their equivalents—for systems with delay in the form $W(p)e^{p\tau}$ ⁽⁶⁾, and for discrete systems $W^*(p)$ ⁽⁷⁾.

Received
5 VII 1960

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Note: Figure translations are in progress. See original paper for figures.

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