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Abstract

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ON ELECTROSTATIC EFFECTS DURING THE MOTION OF A RAREFIED PLASMA IN AN INHOMOGENEOUS MAGNETIC FIELD

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An inhomogeneous magnetic field $\mathbf{H}(\mathbf{r})$, in which a flow of rarefied plasma is moving, acts to different degrees on the motion of its electron and ion components, which leads to charge separation in regions whose dimensions are determined by the scale L of the inhomogeneity of the magnetic field. The formation of spatially extended volume charges in the plasma gives rise to a jump in the electrostatic potential, whose magnitude depends both on the degree of inhomogeneity of $\mathbf{H}$ and on the physical characteristics of the plasma flow. This nonlinear effect is investigated in the present article for the case when $\mathbf{H} = \mathbf{H}(x)$.

1. Let us consider a plane stationary flow of a two-component electron-ion plasma in the magnetic field

$$\mathbf{H}(x) = H_{0i} + H_{0y}\eta(x)\mathbf{j}, \tag{1}$$

where $\eta(x)$ is a nondecreasing function, $\eta = 0$ for $x \leq -L/2$, $\eta = 1$ for $x \geq L/2$. It is immediately seen from (1) that the equation $\operatorname{div}\mathbf{H} = 0$ is satisfied automatically. A field model close to (1) can be realized, for example, by means of a system of steady currents flowing in the direction of the z-axis along plane fixed grids placed in a homogeneous field $\mathbf{H}_0$ parallel to the plane $x = 0$. We shall assume that collisions of charged particles with the grid may be neglected and that the plasma flow interacts only with the magnetic field.

Let $\mathbf{V}$ be the macroscopic velocity of the flow directed along the magnetic field at $x = -\infty$; n_i and n_e are the densities of ions and electrons in the incident (index +) flow, with

$$n_i^+(-\infty) = n_e^+(-\infty) = n_0 \tag{2}$$

(the condition of electrical neutrality). We consider the plasma sufficiently rarefied, so that the inequality

$$l_{i,e} \gg L \gg R_{i,e}, \tag{3}$$

holds, where l and R are, respectively, the mean free path and the Larmor radius of a charged particle. Fulfillment of (3) makes it possible, neglecting collisions, to describe the behavior of the plasma by means of an approximate, so-called "drift" kinetic equation [1,2] for the distribution function of ions (electrons) $f_{i,e}(\mathbf{r}, v, \mu)$, which in our case gives for each component all three integrals of motion:

$$\mu = \frac{v_\perp^2}{2\mathcal{H}} = \text{const}; \qquad \frac{mv^2}{2} + e\varphi + m\mu\mathcal{H} = \mathcal{E} = \text{const}; \qquad \mathcal{A} = \text{const}. \tag{4}$$

Here $\operatorname{rot}\vec{\mathcal{A}} = \vec{\mathcal{H}} = \mathbf{H} + \mathbf{h}$, where $\mathbf{h}$ is the magnetic field associated with drift currents in the plasma; v and $v_\perp$ are the components of the particle velocity parallel and perpendicular to $\vec{\mathcal{H}}$; m, e, μ are the mass, charge, and magnetic moment of the ion (electron); φ is the electrostatic potential. Below we consider flows for which, at $x = -\infty$,

$$\frac{\partial f}{\partial y} = H_0 \frac{\partial f}{\partial \mathcal{A}} = 0, \tag{5}$$

which, in combination with (1), ensures that (5) is satisfied throughout space. If the form of the functions f_i and f_e is known at $x = -\infty$, i.e., if the velocity distribution of particles in the incident flow is specified, then the relations (4) determine f_i and f_e, as solutions of the drift kinetic equation, everywhere; here $\vec{\mathcal{H}}$ and φ must satisfy Maxwell's equations for the self-consistent electromagnetic field [2]. Solving the resulting system of nonlinear differential equations for $\varphi(x)$ and $\mathbf{h}(x)$ involves considerable mathematical difficulties. For a whole range of questions, however, what is of interest is not the exact profile of the potential, but its jump $\delta\varphi = \varphi(\infty) - \varphi(-\infty)$, established as a result of the interaction of the plasma flow with the inhomogeneity $\mathbf{H}$. This makes it possible, using the planar character of the magnetic-field inhomogeneity, to simplify the problem substantially, reducing it to the solution of a system of transcendental equations for the asymptotic values of φ and $\mathbf{h}$.

Taking into account (1), (2), (5), and using the known results of electrostatics for a plane layer, we obtain:

$$\varphi(-\infty) = 0, \qquad \varphi(\infty) = \varphi^1; \qquad -h_y(-\infty) = h_y(\infty) = h_1.$$

(Here and below, by the indices 0, 1 we denote the values of quantities respectively at $x = \mp\infty$.) The equations for the "jumps" φ^1 and h_1 are obtained from the conditions of quasineutrality of the transmitted and reflected plasma flows at $x = \pm\infty$,

$$n_{i1}^+ = n_{e1}^+, \qquad n_{i0}^- = n_{e0}^- \tag{6}$$

and the continuity of the xy-component of the momentum-flux density of the "particles-field" system [3]

$$\left(T_{xy}^{(\text{ch})} + T_{xy}^{(h)}\right)_0 = \left(T_{xy}^{(\text{ch})} + T_{xy}^{(h)}\right)_1. \tag{7}$$

The latter equality is a consequence of the stationarity of the regime under consideration and of the constancy of the currents creating the field **H**. As is easily shown, under these conditions the interaction of the drift currents in the plasma with H_y changes only the xx-component of the momentum flux of the system.

Let us note that from (6) there follows the equality of the macroscopic velocities of the ion and electron components at $x = \infty$ ($V_{i1}^+ = V_{e1}^+$).

Expanding (6), (7), passing to the dimensionless variables $w = v_0/V$ and $q = m\mathcal{H}_0\mu/\theta_0^\perp$, and extracting dimensionless parameters, we obtain equations for $\psi^1 = 2e\varphi^1/m_{iV}^2$ and h_1:

$$\int_0^\infty \int_0^k (f_{i1}^* - f_{e1}^*)\, dq\, dw = 0; \tag{6'}$$

$$\frac{1}{2A^2 n_0} \int_0^\infty \int_0^k \left\{ \frac{H_0 h_1}{H_0^2 + h_1^2}(f_{i0} + \varepsilon f_{e0}) - \frac{H_0(H_{0y} + h_1)}{\sqrt{(H_0^2 + h_1^2)\,[H_0^2 + (H_{0y} + h_1)^2]}}(f_{i1}^* + \varepsilon f_{ei}^*) \right\} w^2\, dq\, dw = \frac{h_1}{H_0}, \tag{7'}$$

where

$$f_i^* = \frac{f_{i0}}{\sqrt{w^2 - \psi - \tau(s-1)q}}, \qquad f_e^* = \frac{f_{e0}}{\sqrt{w^2 + \psi/\varepsilon - \theta(s-1)q/\varepsilon}}.$$

and f^* is normalized according to the condition

$$\left[s \int_0^\infty \int_0^\infty f_i^*\, dq\, dw \right]_0^+ = \langle n \rangle_0^+ = n_0.$$

Here $\varepsilon = m_e/m_i$, $\theta = \theta_{e0}^\perp/\theta_{i0}^\perp$ is the ratio of the "transverse temperatures" of electrons and ions, $\tau = 2\theta_{i0}^\perp/m_{iV}^2$; $s = \mathcal{H}/\mathcal{H}_0$; $A = \sqrt{H_0^2/4\pi m_i n_0 V^2}$ is the so-called Alfvén number.

Fig. 1
Fig. 1

Fig. 2
Fig. 2

Fig. 3
Fig. 3

The parameter k is related to the flux transmission coefficient

$$K = \frac{\langle nv\rangle_0^+ - \langle nv\rangle_0^-}{\langle nv\rangle_0^+} = \int\limits_0^\infty \int\limits_0^k f_0 \, dq \, dw.$$

System (6′), (7′) is closed in the case of complete transmission; in this case $k = k_{\max}$ and $K(k_{\max}) = 1$.

2. The system (6′), (7′) is considerably simplified if

$$A \gg 1. \tag{8}$$

From (7′) we obtain

$$h_1 \simeq \frac{1}{A^2} H_{0y} \left(\frac{H_0}{H_1}\right)^2 \frac{\langle mnv^2\rangle_i^+ + \langle mnv^2\rangle_e\rangle_1}{m_i n_0 V^2} \ll H_{0y},$$

i.e. $\mathscr{H} \simeq H$. ψ^1 is found from (6′), in which $s^1 = H_1/H_0$.

3. As a simple illustration of the application of (6′), let us consider the flow through the field (1) of a packet of particles with $f(-\infty)$, "smeared" over transverse velocities: $(f_{i,e})_0 = n_0 \delta(w-1)$ for $0 \le q \le 1$, $(f_{i,e})_0 = 0$ for $q > 1$. Here the choice of the δ-function corresponds to the assumption that $\theta_\parallel \ll mv^2/2$. When (8) is satisfied, equation (6′) reduces to the equation

$$\frac{\theta}{\varepsilon}\left(\sqrt{1-\psi^1} - \sqrt{1-\psi^1 - \tau k(s^1-1)}\right) =$$

$$= \sqrt{1 + \frac{\psi^1}{\varepsilon}} - \sqrt{1 + \frac{\psi^1}{\varepsilon} - \frac{\theta}{\varepsilon}\tau k(s^1-1)},$$

from which we find

$$\psi^1 = \psi^1(\varepsilon, \theta, \tau k(s^1-1))$$

(see Fig. 1).

The thickened part of the curve corresponds to complete transmission of the flux ($k = K = 1$). When $s^1 > s^1_{\text{cr}}$, reflection begins ($k < 1$). As $k \to 0$,

$$\psi^1 \to \psi^1_{\max} = \frac{\theta^2 - \varepsilon^2}{\theta^2 + \varepsilon}.$$

The potential jump in the inhomogeneous field H leads to an electrostatic shift of the critical value $s^1_{\text{cr}} = (H_1/H_0)_{\text{cr}}$ (Fig. 2).

As the calculation shows,

$$\theta_1^{\parallel} = \frac{\langle m(v - V_1)^2 n\rangle_1^+}{\langle n\rangle_1^+} \quad \text{and} \quad \theta_1^{\perp} = \frac{\langle m\mu\mathcal{H} n\rangle_1^+}{\langle n\rangle_1^+},$$

the "dragging" of electrons by ions through the inhomogeneous magnetic field causes strong adiabatic transverse heating of the light electron component at the expense of the kinetic energy of the ions (4) (Fig. 3). The resulting

anisotropy of the temperatures may lead to the development of an oscillatory instability [(5)] with dissipation of energy.

The electrostatic effects indicated above have been obtained for a special choice of the inhomogeneity **H**, which permits a mathematical investigation of the problem. However, the charge-separation mechanism considered is sufficiently general, and the phenomena associated with it should be present, to one degree or another, in the motion of a rarefied plasma in any inhomogeneous magnetic field. Electrostatic effects may play an essential role in the interaction of streams of conducting gas with the magnetic fields of cosmic objects, where the fulfillment of (3) is usual. In particular, the self-consistent jump of the potential in the stream provides the mechanism, indicated by V. I. Veksler [(4)], for the generation of relativistic electrons in cosmic space.

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