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## Abstract

## Full Text



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# On the Quantum Theory of Unstable Elementary Particles

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**Physics**

1. Unstable states of a physical system, in particular unstable elementary particles, differ in that, in principle, they do not have a definite value of the energy (mass), unlike stable states (particles), and are characterized by the density of the energy distribution (mass distribution)* $\tilde{\omega}(E)$, which, on the basis of the Fock–Krylov theorem ([1]), uniquely determines the decay law $L(t) = |p(t)|^2$ of an unstable physical system:

$$p(t) = M(t)e^{iN(t)} = \int_0^\infty e^{-iEt}\tilde{\omega}(E)\,dE. \tag{1}$$

A description of unstable states of a physical system by means of unstable elementary particles (in "particle" language), from this point of view, in fact means a parametrization of the energy distribution (mass distribution) $\tilde{\omega}(E)$ and the characterization of unstable elementary particles (along with other quantum numbers: charge, spin, etc.) by the parameters of this distribution**. The parametrization may be carried out either by specifying the moments of $\tilde{\omega}(E)$ (see, for example, ([5])), or by specifying the analytic structure of $\tilde{\omega}(E)$ (see, for example, ([1,6])).

Information about unstable physical systems (particles) can be obtained from independent physical experiments: either from the decay law of an unstable physical system, or from experiments on the scattering of decay products by one another, where unstable states (particles) appear as resonances, or from experiments on the scattering (interaction) of unstable physical systems (particles) with other physical systems (particles). As can be shown in a consistent nonstationary scattering theory of unstable particles, the usually measured asymptotic scattering cross section depends on the first moment ( "mass" ) of the energy distribution.

Ordinary unstable elementary particles, decaying by means of the weak interaction, whose lifetimes are of the order of $10^{-8}$–$10^{-10}$ sec., are described by a two-parameter distribution—the dispersion distribution***:

$$\tilde{\omega}(E) = C\frac{1}{(E-E_0)^2+\Gamma^2}, \qquad E \geqslant 0; \tag{2}$$

$C$ is a normalization constant, $\Gamma/E_0 \sim 10^{-15}$ for known (ordinary) unstable particles. $E_0$ is given the meaning of the "mass" of the unstable particle****,

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* For the sake of generality we shall everywhere use the energy distribution, and not the mass distribution, which are obviously related, in order to include in the consideration also unstable states of atoms, nuclei, etc.

** An analogous point of view is in fact also contained in ([2–4]).

*** We do not dwell on the description of $K_0$-, $\bar{K}_0$-mesons, whose mass distribution reduces to (2), but from a principled point of view is treated analogously.

**** In fact, as has already been pointed out, the experimental "mass" is determined by the first moment of $\tilde{\omega}(E)$, which probably exists if one takes into account the "preparation" function and a detailed description of the measuring process (see, for example, ([2,4])); however, owing to the smallness of $\Gamma/E_0$, the first moment of $\tilde{\omega}(E)$ practically reduces to $E_0$.

and $\Gamma$ is associated with the law of its decay. It is not difficult to see that, because of the extreme smallness of $\Gamma/E_0 \sim 10^{-25}$, ordinary experiments (with present-day accuracy) are not sensitive to the "preparation" function $\varphi(E)$, and hence also not to the parameters of the "preparation" function:

$$\tilde{\omega}(E) = \frac{\varphi(E)}{(E-E_0)^2+\Gamma^2}, \tag{3}$$

which justifies the two-parameter approximation (2). The "preparation"function $\varphi(E)$, having no pole singularities in the plane of the complex $E$, characterizes the fine details of the unstable state (its model), while $E_0$ and $\Gamma$, i.e. the position of the pole, depend little on these details.*

Indeed, in experiments on the scattering of unstable particles, the determining quantity is the "mass"—the first moment of $\tilde{\omega}(E)$, which, owing to the smallness of $\Gamma/E_0 \sim 10^{-15}$, reduces to $E_0$ and depends neither on $\Gamma$ nor on $\varphi(E)$; while in decay experiments the observed principal exponential term ([1,2,8,9]) is determined only by $\Gamma$** and, in turn, depends neither on $E_0$ nor on $\varphi(E)$. Experiments on resonant scattering of decay products, because of the weakness of the interaction, are practically unrealizable at the present level of accuracy.

However, the polynomial correction terms to the exponential law of decay ([2,4,7–13]) are very sensitive to $\varphi(E)$ (and to $E_0$), and thus a study of the decay law with considerably greater accuracy could in principle make it possible to determine the parameters of the "preparation" function. Naturally, a more accurate study of scattering, in particular resonant scattering, would also make it possible to determine the "preparation" function; and consequently, for such more accurate experiments, the characterization of unstable elementary particles by means of only two (pole) parameters $E_0$ and $\Gamma$ would prove clearly insufficient, and we would have to refine the concept of an unstable elementary particle by describing it with a larger number of parameters. Thus we arrive at the natural conclusion: **the concept of an unstable elementary particle depends on what experiments, and with what accuracy, we use for its definition.** A substantial refinement of the experiment would thereby make it possible, in particular, to distinguish from one another (as different) elementary particles which at present we regard as identical, i.e. whose parameters $E_0$ and $\Gamma$ coincide.

Of course, for ordinary unstable particles, because of the extreme smallness of $\Gamma/E_0 \sim 10^{-15}$, this refinement of the concept of an elementary particle is of principled, but mainly academic, interest. However, for very short-lived particles, indications of whose existence have appeared in recent experiments ([14])***, for which $\Gamma/E_0 \sim 10^{-1}$, this refinement may prove significant already at the present time.

2. In connection with the possibility (and necessity) indicated above of a principled refinement of the concept of an unstable elementary particle, further study of the inverse problem in the theory of decay, posed and considered earlier ([2,3,8,9]), is of substantial interest. In particular, the following problem is of interest: despite the non-uniqueness of the reconstruction of the energy distribution (mass distribution) from a given decay law, are the poles of $\tilde{\omega}(E)$, and consequently the usual characteristics of unstable elementary particles $E_0$ and $\Gamma$, reconstructed uniquely?

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* See, for example, the study of the $\alpha$-decay model in ([1]) and the study of an unstable $V$-particle in the Lee model ([7]).

** And not by the second moment of $\tilde{\omega}(E)$ ([5]) (see the study carried out in ([1])).

*** Prof. Ya. A. Smorodinskii drew my attention to these works.

3. The problem of reconstructing the energy distribution (mass distribution) $\tilde{\omega}(E)$ from the decay law $L(t) = |p(t)|^2$,

$$\tilde{\omega}(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} M(t) e^{iN(t)} e^{iEt} \, dt \tag{4}$$

reduces to determining the phase $N(t)$ from the modulus $M(t)$. As was shown in previous papers ([2,3,8,9]), a consequence of the spectrality principle is the dispersion relation

$$N(t) = \frac{2t}{\pi} P \int_0^\infty \frac{\log M(t') - \log M(t)}{{t'}^2 - t^2}\, dt' + 2\sum_k \left[ \operatorname{arc\,tg} \frac{\beta_k}{t-\alpha_k} + \operatorname{arc\,tg} \frac{\beta_k}{t+\alpha_k} \right] \equiv$$

$$\equiv N_M(t) + N_k(t), \qquad \beta_k > 0, \tag{5}$$

where $N_M(t)$ is the phase corresponding to the given decay law under the assumption that $p(t)$ has no zeros, while $t_k = \alpha_k + i\beta_k$ are possible zeros of $p(t)$.

Substituting (5) into (4) and applying the convolution theorem, we obtain

$$\tilde{\omega}(E) = \int_{-\infty}^{\infty} \tilde{\omega}_M(E')\tilde{\omega}_k(E-E')\, dE', \tag{6}$$

where

$$\tilde{\omega}_M(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iEt} M(t) e^{iN_M(t)}\, dt, \tag{7}$$

$$\tilde{\omega}_k(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iEt} \prod_k \frac{(t-\alpha_k+i\beta_k)(t+\alpha_k+i\beta_k)}{(t-\alpha_k-i\beta_k)(t+\alpha_k-i\beta_k)}\, dt.$$

It is clear from (6) that it is sufficient to restrict ourselves to the consideration of one pair of zeros, since the inclusion of a larger number of zeros can be obtained from recurrence relations. Taking into account only one pair of zeros and separating out the delta function in $\tilde{\omega}_k(E)$, after evaluation we obtain

$$\tilde{\omega}(E) = \tilde{\omega}_M(E) +$$

$$+ \int_{-\infty}^{E} \tilde{\omega}_M(E')\, 4\beta_k e^{-(E-E')\beta_k} \left[ \frac{\beta_k}{\alpha_k} \sin(E-E')\alpha_k - \cos(E-E')\alpha_k \right] dE'. \tag{8}$$

The explicit expression thus obtained for $\tilde{\omega}(E)$ in terms of $\tilde{\omega}_M(E)$ and the parameters $\alpha_k, \beta_k$ of the possible zeros of $p(t)$ makes it possible to prove the following lemmas:

**Lemma 1.** The singularities of $\tilde{\omega}(E)$ as a function of the complex variable $E$ (singularities at finite distance in the $E$-plane) can be located only at those points where $\tilde{\omega}_M(E)$ has singularities.

**Lemma 2.** The poles of $\tilde{\omega}(E)$ coincide with possible poles of $\tilde{\omega}_M(E)$. At those points $E = E_0 \pm i\Gamma$ at which $\tilde{\omega}_M(E)$, and consequently also $\tilde{\omega}(E)$, has poles, $\tilde{\omega}(E)$ also has a logarithmic singularity.

The proof of Lemma 1 is evident from the definition of $\tilde{\omega}(E)$ according to (8).

The proof of Lemma 2 in fact reduces to investigating the behavior of a Cauchy-type integral at the ends of the path of integration ([15]), and can also be obtained by directly separating out the singularity in the integral term of (8). It is important to emphasize that, as a concrete investigation shows, the logarithmic singularity certainly exists for $\tilde{\omega}(E)$, since the coefficient of the logarithmic singularity is by no means equal to zero for any positions of the pole of $\tilde{\omega}_M(E)$.

It is not difficult to see that taking into account the remaining possible zeros of $p(t)$ does not alter the formulated lemmas.

4. Relying on the lemmas indicated above, we arrive at the following main result, which we formulate as a theorem:

**Theorem.** *The problem of reconstructing the pole singularities of the energy distribution (mass distribution) $\tilde{\omega}(E)$ as a function of the complex variable $E$ from the given decay law $L(t) = |p(t)|^2$ is unique. The poles of $\tilde{\omega}(E)$ coincide with the poles of $\tilde{\omega}_M(E)$:*

$$\tilde{\omega}_M(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{iEt} M(t) \exp\left[ i\frac{2t}{\pi} P \int_0^{\infty} \frac{\log M(t') - \log M(t)}{t'^2 - t^2}\, dt' \right] dt. \quad (9)$$

Thus, we arrive at the following conclusion: the pole parameters $E_0$ and $\Gamma$, which describe ordinary unstable particles, are uniquely determined from the given decay law and, consequently, the study of the decay law makes it possible to obtain complete information about the mass distribution of ordinary unstable particles. The possible zeros of the amplitude $p(t)$, however, affect only the "preparation" function and are essential only for the description of unstable elementary particles (short-lived ones) for which the usual two-parameter description is insufficient. In this case, for such particles, the study of the decay law alone does not give a complete description of the mass distribution, since the decay law is insensitive to logarithmic singularities.

According to Lemma 2, we arrive at the result that completes the investigation of the decay law of ordinary unstable particles: the function $p(t)$ corresponding to the dispersion distribution (2) has no complex zeros. Indeed, suppose the contrary. Then it would follow from Lemma 2 that the dispersion distribution (2) has a logarithmic singularity, which is false.

5. Let us make several remarks concerning experimental problems. The fundamental experimental problem consists in discovering unstable elementary particles for which the usual two-parameter (pole) description is insufficient. For this purpose, the most promising approach at present is a careful study of the detailed resonance shape in experiments analogous to [14]. No less interesting is the following problem: the search for such particles whose ordinary parameters $E_0$ and $\Gamma$ coincide, while the parameters

connected with the "preparation" function are different. For this purpose, a detailed study of the resonance shape in various possible decay channels of the unstable particle is promising.

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*Note: Figure translations are in progress. See original paper for figures.*