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Abstract

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ON THE DISTRIBUTION OF THE DENSITY OF CHARGED PARTICLES IN METEOR TRAILS

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To study the reflection of radio waves from a meteor trail it is necessary to know the distribution in it of the density of charged particles. With time the density changes because of the processes occurring in the trail: diffusion, recombination, and attachment of electrons to neutral oxygen atoms and molecules. The last process does not directly change the density distribution and affects only the effective recombination coefficient. As for the first two, their relative influence on the particle-density distribution is determined by the dimensionless parameter

$$\varepsilon = \alpha q^*/D$$

(α is the recombination coefficient, q is the number of ionizations produced by the meteor per unit length of the trail, and D is the diffusion coefficient), expressing the ratio of the rate of change of particle density due to recombination to the rate of its change due to diffusion. According to ([1]), the diffusion coefficient is

$$D \simeq \frac{1}{3} \frac{\bar{v}}{Q_d(n + n_M)}, \tag{1}$$

where $\bar{v}$ is the mean velocity of the ions; Q_d is the diffusion cross section; n and n_M are the densities of charged and neutral particles, respectively. Under the conditions of interest to us (height 90-100 km), $n \sim 5 \cdot 10^{13}$ cm^{-3}, $\bar{v} \sim 5 \cdot 10^4$ cm/sec, $Q_d \sim 10^{-13}$ cm^2, so that $D \sim 3 \cdot 10^4$ cm^2/sec, which is also consistent with the experimental data given in ([3]). According to ([2]), the mean effective recombination coefficient at heights of 90-100 km is $\sim 10^{-8}$ cm^3/sec. Thus, values $\varepsilon \sim 1$ correspond to linear ion densities $q \sim 10^{13}$ cm^{-1}, which appears quite realistic. With decreasing height, ε increases rapidly, so that the need to take recombination into account at heights below 100 km seems obvious.

Let us consider a meteor moving uniformly with velocity v along the z-axis. The density of the charged particles it creates is described by the equation*

$$\frac{\partial n}{\partial t} = D\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial n}{\partial r}\right) - \alpha n^2 + \frac{q}{2\pi v}\,g\left(\frac{r}{r_0}\right)\delta(z - vt) \tag{2}$$

under the conditions

$$n(0,t) < \infty; \qquad n(\infty,t) = 0. \tag{3}$$

The function $g(r/r_0)$ entering into (2) differs appreciably from zero only in the region $r < r_0$ (r_0 is a length of the order of the meteor' s dimensions) and is such that

$$\frac{1}{r_0^2}\int_0^\infty g\left(\frac{r}{r_0}\right) r\,dr = 1.$$

* Diffusion along the direction of motion can obviously be neglected.

We note that, for a solution of the boundary-value problem (2)–(3) to exist, the function $g(r/r_0)$ must be bounded from above.

If the parameter ε is sufficiently small, recombination may be taken into account as a perturbation. Without presenting the simple calculations here, we write the first terms of the expansion in powers of ε:

$$W(\rho,\xi) = \varepsilon f(\rho,\xi) + \frac{\varepsilon^2}{2}\int_0^\xi \frac{d\xi'}{\xi - \xi'}\left[\int_0^\infty \exp\left\{-\frac{\rho^2 + \rho {o'}^2}{4(\xi - \xi')}\right\} I_0\left(\frac{\rho\rho'}{2(\xi - \xi')}\right)\right.$$
$$\left.\times f^2(\rho',\xi')\,\rho'\,d\rho'\right] \qquad (\xi > 0), \tag{4}$$

$$W(\rho,\xi) = 0 \qquad (\xi < 0),$$

where

$$f(\rho,\xi) = \frac{1}{4\pi\xi}\int_0^\infty g(\rho')\exp\left[-\frac{\rho^2 + \rho {o'}^2}{4\xi}\right] I_0\left(\frac{\rho\rho'}{2\xi}\right)\rho'\,d\rho';$$

$$W(\rho,\xi) = \frac{\alpha r_0^2}{D}\,n(r,t), \qquad \rho = \frac{r}{r_0}, \qquad \xi = \frac{D}{v r_0^2}(vt - z).$$

However, if the parameter ε is not small, recombination cannot be regarded as a correction, and the perturbation method is inapplicable. We shall obtain an approximate solution of the problem in this case.

It is convenient to transform the boundary-value problem (2)–(3) somewhat, by converting the source into an initial condition:

$$\frac{\partial W}{\partial \xi} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial W}{\partial \rho}\right) - W^2; \tag{5}$$

$$W(0,\xi) < \infty, \qquad W(\infty,\xi) = 0; \tag{6}$$

$$W(\rho,0) = \frac{\varepsilon}{2\pi}\, g(\rho). \tag{6'}$$

As the function in (6′) we choose

$$g(\rho) = 2e^{-\rho^2}, \tag{7}$$

which satisfies the conditions mentioned above. Put in equation (5)

$$W(\rho,\xi) = \frac{4y(x,\xi)}{4\xi+1}, \qquad x = \frac{\rho^2}{4\xi+1}. \tag{8}$$

For the function $y(x,\xi)$ one obtains the equation

$$x\frac{\partial^2 y}{\partial x^2} + (1+x)\frac{\partial y}{\partial x} + y = y^2 + \left(\xi + \frac{1}{4}\right)\frac{\partial y}{\partial \xi}, \tag{9}$$

which is equivalent to the following system of ordinary differential equations for the successive moments of the function $y(x,\xi)$:

$$\left(\xi + \frac{1}{4}\right)\frac{dM_n}{d\xi} = n^2 M_{n-1} - nM_n - N_n, \tag{10}$$

where

$$M_n = \int_0^\infty y(x,\xi)x^n\,dx, \qquad N_n = \int_0^\infty y^2(x,\xi)x^n\,dx \qquad (n = 0,1,...).$$

We shall construct a function $y(x,\xi)$ satisfying the first two equations of system (10). Such a function must correctly describe the time dependence of the number of particles in the cross section of the trail perpendicular to the Z axis, and the change with time of the mean size of the trail.

We shall seek an approximate solution in the form

$$y(x,\xi) = h(\xi)e^{-xp(\xi)}, \tag{11}$$

where h and p are unknown functions of ξ. Substituting (11) into (10), we obtain the system of equations

$$\left(\xi + \frac{1}{4}\right)\frac{d}{d\xi}\left(\frac{h}{p}\right) = 1 - \frac{h^2}{2p},$$

$$\left(\xi + \frac{1}{4}\right)\frac{d}{d\xi}\left(\frac{h}{p^2}\right) = \frac{h}{p} - \frac{h}{p^2} - \frac{h^2}{4p^2}, \tag{12}$$

the initial conditions for which, $h(0) = \varepsilon/4\pi,\ p(0) = 1$, follow from (7).

The system (12) is readily integrated, and the solution has the form

$$\int\limits_u^{\varepsilon/4\pi} e^{2/u}u^{-5/2}\,du = 4\sqrt{\frac{\pi}{\varepsilon}}\,e^{8\pi/\varepsilon}\xi;$$

$$p = \frac{2}{4\xi+1}\sqrt{\frac{\pi}{\xi}}\sqrt{u}\,e^{8\pi/\varepsilon-2/u};$$

$$h = \frac{2\xi}{4\xi+1}\sqrt{\frac{\pi}{\xi}}\,u\sqrt{u}\,e^{8\pi/\varepsilon-2/u}. \tag{13}$$

Formulas (4), (8), (11), and (13) solve the problem posed.

We note that $u(\xi)$ is proportional to the total number of particles in a cross section of the trail perpendicular to the Z axis.

For small times ($\xi \ll 2\pi/\varepsilon$), for $u(\xi)$ we obtain:

$$u(\xi) = \frac{\varepsilon}{4\pi} - 2\left(\frac{\varepsilon}{4\pi}\right)^2\xi. \tag{14}$$

For sufficiently large times, such that $u \ll \varepsilon/4\pi$, we have asymptotically:

$$\frac{e^{2/u}}{\sqrt{u}}\left(1 - \frac{u}{4} - \frac{u^2}{8}\sum_{n=0}^{N}\frac{(2n+1)!!}{2^{2n+1}}u^n\right) \sim 8\sqrt{\frac{\pi}{\varepsilon}}\,e^{8\pi/\varepsilon}\xi, \tag{15}$$

where $N \ll 2/u$.

It follows from (15) that as $\xi \to \infty$ the solution becomes (up to logarithmic factors) dependent only on x.

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Note: Figure translations are in progress. See original paper for figures.