



Soviet-era science, translated into English

Physics

A. L. POLYAKOVA

1961

SovietRxiv

View the original and related papers at <https://sovietrxiv.org/items/ru-196101.55348>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

Physics

A. L. POLYAKOVA

A PLANE SOUND WAVE OF FINITE AMPLITUDE IN A MOVING MEDIUM

(Presented by Academician N. N. Andreev, 22 XII 1960)

The Riemann solution describing the propagation of a plane sound wave of finite amplitude can be generalized to the case of a medium moving with constant velocity V . For definiteness, let us assume that the velocity V is directed toward positive values of x (where x is the coordinate in a fixed frame of reference), while the sound wave propagates along the flow or against it, depending on whether the observer is to the right or to the left of the source. This latter circumstance will be indicated by the plus or minus sign before c_0 , the speed of sound in the undisturbed medium.

Let us consider three cases:

1. The observer is stationary relative to the source, which is located at the point $x = 0$, while the medium moves relative to both of them with velocity V (wind). In the coordinate system associated with the flow ($x' = x - Vt$), the fluid is at rest and, consequently, the solution of the hydrodynamic equations is relation (1),

$$x' = t[v \pm c(v)] + f(v), \quad (1)$$

the so-called Riemann solution. Here $f(v)$ is an arbitrary function of the oscillatory particle velocity v , which is determined from the boundary conditions; $c(v)$ is the speed of sound, which in the nonlinear case depends on v . For the adiabatic equation of state it can be shown that

$$c(v) = c_0 \pm \frac{\gamma - 1}{2} v, \quad (2)$$

where γ is the ratio of heat capacities. Passing in (1) to the fixed coordinate system (x, t) , taking into account (2) and the boundary condition: at $x = 0$, $v = F(t)$, we obtain

$$v = F\left(t - \frac{x}{V \pm c_0 + \varepsilon v}\right), \quad (3)$$

where $\varepsilon = (\gamma + 1)/2$.

From this expression it is clear that the presence of a flow leads to drift of the sound wave in the direction of the wind. If the sound wave propagates against the flow, the minus sign must be taken before c_0 . When $V - c_0 = 0$, expression (3) takes the form

$$v = F\left(t - \frac{x}{\varepsilon v}\right). \quad (4)$$

Analysis of expression (4), for example for the case when a sinusoidally oscillating source is located at the origin, shows that the function

$v(x, t)$, determined by equation (4), is multivalued for any values of x and t , i.e., this expression does not describe any real physical process. As is known, the Riemann solution in a stationary medium becomes ambiguous only beginning at some distance, depending on the Mach number $M = v/c_0$ and on the frequency. At this distance a periodic shock wave is formed. In the presence of a flow with velocity $V = c_0$, directed toward the sound, one may assume that the shock wave is formed at $x = 0$, at the emitter itself, but does not propagate farther, since the propagation velocity $V - c_0 = 0$.

For clarification, let us turn to the solution of the problem in the second approximation. Consider the case $V < c_0$. Then, in a moving medium, the role of the Mach number for sound should be played by the ratio $v/(V \pm c_0)$. Therefore, in the case when sound propagates against the flow of the liquid and V is close to c_0 , it may turn out that, although $v \ll c_0$, the ratio $v/(V - c_0)$ is not a small quantity. If we assume that at $x = 0$, $v = A \sin \omega_0 t$, then in the first approximation the solution has the form $v_1 = A \sin(\omega_0 t - kx)$, where $k = \omega_0/(V \pm c_0)$, and in the second approximation

$$v_2 = \frac{A^2 \omega_0 \varepsilon}{2(V \pm c_0)^2} x \sin 2(\omega_0 t - kx). \quad (5)$$

Consequently, as $V \rightarrow c_0$, this solution becomes inapplicable for arbitrarily small x . The distance to the point where the sound wave turns into a periodic shock wave is equal to

$$x_n = \frac{(V \pm c_0)^2}{\varepsilon A \omega_0} \quad (6)$$

(expression (6) is obtained from the condition that, in a sawtooth wave, the ratio of the amplitudes of the first and second harmonics is equal to 2), i.e., the distance to the formation of the shock wave depends on the flow velocity V , and for $V - c_0 = 0$, $x_n = 0$, as was already noted above.

Taking into account the viscosity of the medium leads to the fact that, in the presence of a flow, one cannot use the accepted expression for the Reynolds

number—the parameter characterizing the relation between viscosity and non-linearity. The Reynolds number is the ratio of two dimensionless parameters $\chi = b\omega/\rho_0 c_0^2$ and $M = v/c_0$, where $b = \eta + \frac{4}{3}\zeta$ is the viscosity, i.e., $\text{Re} = \chi/M = p/b\omega$, where p is the amplitude of the variable sound pressure. In a moving medium the absorption will not change, while the number M will be equal to $v/(V \pm c_0)$; consequently, the Reynolds number $\text{Re}' = \frac{c_0}{V \pm c_0} \text{Re}$. It is clear, however, that the influence of the factor $c_0/(V \pm c_0)$ is substantial only at velocities comparable with the speed of sound in the medium. The presence of viscosity in the medium, as is known, leads to the fact that at distances far from the emitter the high-frequency harmonics are absorbed and the wave again becomes practically close to sinusoidal, but already of small amplitude.

2. In the case of a source moving with constant velocity V in the positive direction of the x -axis and emitting a harmonic wave whose frequency in the coordinate system associated with the source is equal to ω_0 , the Riemann solution has the form

$$v = A \sin \omega_0 \left(t - \frac{x - Vt}{\pm c_0 - V + \varepsilon v} \right). \quad (7)$$

The upper sign here corresponds to the source approaching the observer, and the lower sign to its receding. In the linear approximation we obtain from this the usual Doppler effect for a moving source

$$v_1 = A \sin(\omega t - kx),$$

where

$$\omega = \omega_0/(1 \mp \beta), \quad \beta = V/c_0, \quad k = \omega/c_0.$$

In the second approximation

$$v_2 = \frac{\omega_0 \varepsilon A^2}{(V \mp c_0)^2} (x - Vt) \sin 2(\omega t - kx). \quad (8)$$

The quantity $|x - Vt|$ is equal to the distance between the source and the observer. For this case one can obtain a relation, analogous to (6), determining the distance from the source to the point where a discontinuity is formed. If x —the position of the observer—is fixed, then this relation will determine the instants at which the observer receives a wave of sawtooth form.

It is evident from the formulas that the character of the received signal (its spectrum) depends essentially on the distance to the source. If the viscosity of the medium is taken into account, it will turn out that the observer will receive

the following picture. When the source, approaching from the side of negative values of x , is very far away—at distances much greater than $(V - c_0)^2/\varepsilon\omega_0 A$ —the observer receives a wave of small amplitude, practically sinusoidal, with frequency ω . As the source approaches, the wave amplitude grows and the relative weight of the high-frequency harmonics in the signal spectrum increases. At the instant $t_1 = x/V - (V - c_0)^2/\varepsilon\omega_0 AV$ the observer will receive a wave whose form is maximally distorted. With further motion of the source the fraction of high-frequency harmonics will decrease, while the amplitude will continue to grow. After the instant $t_2 = x/V$ the source will move away from the observer and the whole picture will repeat in reverse order, with the only difference that the instant t_3 at which the wave of the most distorted form arrives as the source recedes will be farther from the instant t_2 than the instant t_2 is from t_1 , namely

$$\frac{t_2 - t_1}{t_3 - t_2} = \left(\frac{V - c_0}{V + c_0} \right)^2,$$

i.e., this ratio depends only on the velocity of the source and does not depend on the emitted frequency or amplitude, while the quantity $t_3 - t_1 = 4c_0/\varepsilon\omega_0 A$ does not depend on the flow velocity.

3. The third case is in a certain sense the reverse of the preceding one: the sound source moves together with the fluid flow, while the observer is in a fixed coordinate system. The Riemann solution has the form

$$v = F \left(t - \frac{x - Vt}{\pm c_0 + \varepsilon v} \right).$$

In the case of a harmonic source, the first and second approximations have the form

$$v_1 = A \sin(\omega t - k_0 x), \quad \omega = \omega_0(1 \pm \beta), \quad k_0 = \frac{\omega}{c_0 + V},$$

$$v_2 = \frac{A^2 \omega_0 \varepsilon}{2c_0^2} (x - Vt) \sin 2(\omega t - k_0 x).$$

The picture received by the observer is analogous to that described above, only the time relations have a somewhat different form:

$$(t_3 - t_2)/(t_2 - t_1) = 1, \quad t_3 - t_1 = 2c_0^2/\varepsilon\omega_0 AV.$$

Acoustics Institute
Academy of Sciences of the USSR

Received
20 XII 1960

References

1. L. D. Landau, E. M. Lifshitz, *Fluid Mechanics*, 1954.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.