Mathematics

1961

SovietRxiv

Abstract

Full Text

Mathematics

G. S. Khovanskii

On the Representation of the Equation $f_1+f_2+f_3+f_4+f_5+f_6=0$ by a Nomogram with an Oriented Transparent Overlay in the Form of a Ruler

(Presented by Academician A. A. Dorodnitsyn, 28 I 1961)

For an equation with 6 variables

$$f_1+f_2+f_3+f_4+f_5+f_6=0, \tag{1}$$

where f_1 is an abbreviated notation for the function $f_1(\alpha_1)$, etc., it is possible to construct a convenient and easily adjustable nomogram with an oriented transparent overlay in the form of a ruler. Transform equation (1):

$$f_1+\frac{\frac{\mu'\mu''}{\mu'+\mu''}f_3+\frac{\mu'\mu''}{\mu'+\mu''}f_6+\mu' f_4}{\mu'}=-f_2-\frac{\frac{\mu'\mu''}{\mu'+\mu''}f_3+\frac{\mu'\mu''}{\mu'+\mu''}f_6+\mu'' f_5}{\mu''}, \tag{2}$$

where μ' and μ'' are parameters satisfying the condition $\mu'\mu''(\mu'+\mu'')\neq 0$. Set

$$f_1+\frac{\frac{\mu'\mu''}{\mu'+\mu''}f_3+\frac{\mu'\mu''}{\mu'+\mu''}f_6+\mu' f_4}{\mu'}=-\beta+T_4, \tag{3}$$

$$-f_2-\frac{\frac{\mu'\mu''}{\mu'+\mu''}f_3+\frac{\mu'\mu''}{\mu'+\mu''}f_6+\mu'' f_5}{\mu''}=-\gamma+T_5, \tag{4}$$

where β and γ are auxiliary variables; T_4 and T_5 are arbitrary functions. Then equation (2) is written as

$$\beta-T_4=\gamma-T_5. \tag{5}$$

We bring equations (3), (4), and (5) to the nomographable form [1]:

$$\mu'(-\beta - f_1) - \mu'(f_4 - T_4) = \mu''(\gamma - f_2) - \mu''(f_5 + T_5) =$$

$$= \frac{\mu'\mu''}{\mu' + \mu''} f_3 + \frac{\mu'\mu''}{\mu' + \mu''} f_6, \tag{6}$$

$$\beta - T_4 = \gamma - T_5 = \delta - T_6,$$

where δ is an auxiliary variable; T_6 is an arbitrary function.

The equations of the elements of the nomogram, after the introduction of the transformation parameters a_0, b_0, a'_0, b'_0, a, b, c, d, μ_y, $\delta_x = 0.5(\mu' - \mu'')$, are given in Table 1.

If we set $T_4 = T_5 = T_6 = 0$ and $b'_0 = b = d = 0$, then the scales α_4, α_5, and α_6 will be located on one straight line, and the transparent overlay will have the form of a ruler. From equations (6) we obtain

$$\beta = \gamma = \delta = \frac{-\mu'(f_1 + f_4) + \mu''(f_2 + f_5)}{\mu' + \mu''}. \tag{7}$$

An important feature of the nomogram with an oriented transparent overlay in the form of a ruler for relation (1) is the presence of different moduli of the scales of the variables α_4, α_5, and α_6, with two moduli μ' and μ'' arbitrary,

Table 1

Fixed plane

Coordinates	Field (α_1, β)	Field (α_2, γ)	Field (α_3, δ)
x	$a_0 - 0.5(\mu' + \mu'')\beta - \mu' f_1$	$a_0 + a + 0.5(\mu' + \mu'')\gamma - \mu'' f_2$	$a_0 + c + 0.5(\mu' - \mu'')\delta + \frac{\mu'\mu''}{\mu' + \mu''} f_3$
y	$b_0 + \mu_y \beta$	$b_0 + b + \mu_y \gamma$	$b_0 + d + \mu_y \delta$

Transparent overlay

Coordinates	Scale α_4	Scale α_5	Scale α_6
x	$a'_0 + \mu' f_4 - 0.5(\mu' + \mu'')T_4$	$a'_0 + a + \mu'' f_5 + 0.5(\mu' + \mu'')T_5$	$a'_0 + d - \frac{\mu'\mu''}{\mu' + \mu''} f_6 + 0.5(\mu' - \mu'')T_6$
y	$b'_0 + \mu_{yT}4$	$b'_0 + b + \mu_{yT}5$	$b'_0 + d + \mu_{yT}6$

and the third, $\dfrac{\mu'\mu''}{\mu'+\mu''}$, is determined by the same law as the modulus of the mean scale in nomograms of aligned points with parallel scales. In this case the families of parallel straight lines α_1 and α_2 form equal angles with the family of horizontal straight lines of the auxiliary variable β, independently of the values of μ' and μ''. The family of parallel straight lines α_3 always makes with the lines β a more obtuse angle than do the lines α_1 and α_2.

Fig. 1. a—fixed plane; b—transparent overlay

Figure 1 gives a nomogram for the formula $z = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$. The ranges of variation of the variables are: $0 \leq \alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5 \leq 5$, $0 \leq z \leq 25$. In constructing the nomogram, Table 1 takes $f_1 = \alpha_1$, $f_2 = \alpha_2$, $f_3 = \alpha_3$, $f_4 = \alpha_4$, $f_5 = \alpha_5$, $f_6 = -z$, $T_4 = T_5 = T_6 = 0$, $b_0' = b = d = 0$, $\mu' = \mu'' = \mu_y = 5$ mm, $a_0 = b_0 = 0$, $a = 80$ mm, $c = 115$ mm. The upper and lower directing straight lines correspond to the limiting values of β, namely $\beta_{\min} = -5$ and $\beta_{\max} = 5$, found from formula (7).

If in equations (3)–(6) and in Table 1 one replaces T_4, T_5, and T_6 by the auxiliary variables $\bar{\beta}$, $\bar{\gamma}$, and $\bar{\delta}$, then on the transparent overlay, instead of scales

will be the families of straight lines α_4, α_5, and α_6, parallel respectively to the straight lines α_1, α_2, and α_3 of the fixed plane. The nomogram will have as its elements only families of parallel straight lines.

Let us also note that, by introducing auxiliary variables and arbitrary functions into equation (1), it can be reduced to a nomographable form in four more ways:

1. $(f_1 + f_2 + T_{12}) - \beta = T_{34} - \gamma = (-f_5 - f_6) - 0,$
$T_{12} - \beta = (f_3 + f_4) - 0 = T_{56} - \delta;$

2. $(f_1 + f_2 + T_{12}) - \beta = T_{34} - \gamma = -f_5 - f_6,$
$T_{12} - \beta = (f_3 + f_4) - 0 = \delta - T_6;$

3. $(f_1 + f_2 + T_{12}) - \beta = \gamma - T_4 = -f_5 - f_6,$
$T_{12} - \beta = f_3 + f_4 = \delta - T_6;$

4. $(\beta + f_1) + (f_2 - T_2) = \gamma - T_4 = -f_5 - f_6,$
$\beta - T_2 = f_3 + f_4 = \delta - T_6.$

However, in none of them can the scales of the transparency be arranged on a single straight line.

Computing Center
Academy of Sciences of the USSR

Received
26 I 1961

References

1. G. S. Khovanskii, *Nomograms with an Oriented Transparency*, Moscow, 1957.

Note: Figure translations are in progress. See original paper for figures.