ON FLUCTUATIONS OF ELECTRON DENSITY IN THE IONOSPHERE

Table 1

1961

SovietRxiv

Abstract

Full Text

PHYSICS

E. A. Novikov

ON FLUCTUATIONS OF ELECTRON DENSITY IN THE IONOSPHERE

(Presented by Academician A. N. Kolmogorov on 13 III 1961)

1. We shall consider turbulent fluctuations of electron density (e.d.f.) in the lower ionosphere (more precisely, up to heights of $\sim$ 110 km, where the frequency of collisions of ions with neutral particles is still large in comparison with their Larmor frequency of rotation in the Earth' s magnetic field). The turbulent motion of the gas in the lower ionosphere may be assumed locally isotropic, since the magnetic field has no substantial effect on it ([1]). At the same time, by the method of scattering of ultrashort radio waves it has been found that small-scale e.d.f. are strongly elongated along the direction of the magnetic field, and the degree of anisotropy increases as the wavelength, i.e., the scale of the corresponding fluctuations, decreases ([2]). In ([3]) the conclusion was drawn that locally isotropic turbulence does not lead to e.d.f. possessing such anisotropy (despite the essential role of the magnetic field in the process of formation of e.d.f.). However, this conclusion seems to us to be incorrect. The point is that the consideration in ([3]) was carried out only for scales much larger than the inner scale of turbulence $\lambda_0 = \varepsilon^{-1/4}\nu^{3/4}$ ([4]) (ε is the dissipation of kinetic energy calculated per unit mass of gas, ν is the kinematic viscosity), whereas comparison of the data of ([2]) with Table 1 shows that the dimensions of anisotropic e.d.f. correspond to the dissipation interval (of the order of and smaller than λ_0).

Table 1

Dependence of the inner scale of turbulence λ_0 on the height z in the ionosphere

z, km	50	60	70	80	90	100	110	120	130	140	150
λ_0, m	$8.0 \cdot 10^{-2}$	$1.9 \cdot 10^{-1}$	$4.5 \cdot 10^{-1}$	1.5	5.2	$1.9 \cdot 10^{1}$	$6.0 \cdot 10^{1}$	$2.0 \cdot 10^{2}$	$4.9 \cdot 10^{2}$	$1.4 \cdot 10^{3}$	$3.0 \cdot 10^{3}$

Table 1 was calculated for the value $\varepsilon = 10^3$ erg/g$\cdot$sec, found from experiments on the reflection of radio waves from meteor trails ([5]).* Below we shall establish that locally isotropic turbulence in the dissipation interval creates e.d.f. strongly elongated along the direction of the Earth' s magnetic field. In addition, the spectral function of e.d.f. (which is proportional to the scattering cross section of ultrashort radio waves ([6])) will be expressed by us in the dissipation interval in terms of the spectral density of the kinetic energy of the turbulent motion of the gas. Thus, the possibility is opened for an experimental study of the turbulence spectrum in the dissipation interval, which is a very difficult problem under terrestrial conditions in view of the smallness of λ_0 in the lower atmosphere. In ([7]) a definite formula is proposed for the spectrum of kinetic energy in the dissipation interval, which can be compared with detailed experiments on the scattering of ultrashort radio waves by e.d.f.

2. The equation describing the change of electron density in the ionosphere, in an important limiting case (for $\alpha_i = \omega_i/\nu_i \ll 1 \ll \alpha_e = \omega_e/\nu_e$)

---

* The data of ([5]) refer to a narrower altitude range, 80–100 km; however, ε varies with height slowly in comparison with ν and enters the expression for λ_0 to the power 1/4, therefore a possible deviation of ε by an order of magnitude will not qualitatively change Table 1.

has the form ([3])

$$\frac{\partial N}{\partial t} + (\mathbf{v}\nabla)N - \gamma\Delta N = -\alpha_i \overline{N}(\mathbf{n}\,\mathrm{rot}\,\mathbf{v}) \tag{1}$$

Here N is the electron density; $\overline{N}$ is the mean value; $\mathbf{v}$ is the velocity of gas motion; $\mathbf{n}$ is the unit vector of the Earth' s magnetic field; γ is the coefficient of ambipolar diffusion, approximately equal to 2ν*; ω_i and ν_i are, respectively, the Larmor frequency and the frequency of collisions of ions with neutral particles**, ω_e and ν_e are the same quantities for electrons. Equation (1) is approximately applicable in the altitude range 80–110 km.

One can distinguish two mechanisms for the occurrence of turbulent fluctuations of the electron density in the ionosphere. The first of them is associated with the presence of a vertical gradient of the mean electron density $d\overline{N}/dz$ and turbulent mixing against the background of this gradient, and the second with the Earth' s magnetic field, whose influence in equation (1) is taken into account by the term on the right-hand side. Apart from this additional term, equation (1) is the equation of convection and diffusion of a passive impurity. The question of the structure of small-scale fluctuations of a passive impurity in a turbulent flow was considered in papers ([8],[9]), where, in certain ranges of wave numbers, spectra were calculated for two limiting cases: $k \ll \nu$ and $k \gg \nu$ (k is the diffusion coefficient). Using the results of paper ([7]), one can propose a more general method of calculation, which is valid for $k \sim \nu$. (The method of calculation and the derivation of the formulas given below will not be presented here for lack of space.) From equation (1), under the assumption of local isotropy of the turbulent velocity field, one can obtain the following relation between the three-dimensional spectral function of the electron-density fluctuations $\Phi(p, \mu)$ and the spectral density of kinetic energy $E(p)$:

$$\Phi(p,\mu)=\frac{E(p)}{4\pi(\gamma-\nu)^2p^4}\left[(\overline{N}_1)^2 f(\mu)+\frac{\chi_1}{3\gamma p^2}\right]\qquad (p\lambda_0\gg 1;\ \gamma>\nu);\qquad (2)$$

$$\chi_1=2\gamma\,\overline{(\Delta N_1)^2};\qquad f(\mu)\simeq \alpha_i^2(1-\mu^2)\qquad (\alpha_i\ll 1\ll \alpha_e).\qquad (3)$$

Here p is the wave number; μ is the cosine of the angle between the direction of the wave vector and $\mathbf{n}$; $\overline{N}_1$ denotes the electron-density fluctuations associated with $d\overline{N}/dz$; the bar denotes averaging in the usual theoretical-probabilistic sense; the formula for $E(p)$ is given in ([7]). For the quantity χ_1 one may propose the following approximation:

$$\chi_1\simeq 2hv_z\left(\frac{d\overline{N}}{dz}\right)^2,\qquad (4)$$

where $h = RT/mg$ is the height of a homogeneous atmosphere (R is the gas constant, T the absolute temperature, m the molecular weight of the gas, g the acceleration due to gravity); v_z is the root-mean-square value of the vertical turbulent pulsations of the velocity. The two terms in (2) correspond to the two mechanisms of generation of electron-density fluctuations indicated above; moreover, as the wave number increases, the second term decreases faster than the first, i.e., the degree of anisotropy increases as the scales of the fluctuations decrease.

It follows from (2)-(3) that the spectral function attains its smallest values at $|\mu|\sim 1$ and its largest at $\mu\sim 0$. This means that the electron density changes little along the direction of the field and changes strongly in the perpendicular direction, i.e., the inhomogeneities are elongated along the field.

Using the equations given in ([3]), one can, in the dissipation interval, obtain a more general formula for arbitrary values of the parameter α_e, which differs from (2)-(3) in the form of the function $f(\mu)$. We indicate here an interes-

* In the present note we do not take into account the anisotropy of ambipolar diffusion, which appears beginning at altitudes ~ 100 km.

** A factor 1/2 has been introduced into ν_i, taking into account that an ion, in collision with a neutral particle of equal mass, transfers to it only half of its relative velocity.

opposite limiting case $a_e\ll 1$ (beginning at 65 km and below), for which

$$f(\mu)\simeq \alpha_i^2 a_e^2\mu^2(1-\mu^2)\qquad (\alpha_i, a_e\ll 1).\qquad (5)$$

It follows from this formula that anisotropic electron-density fluctuations are arranged crosswise at an angle of 45° to the field. As far as we know, such cases have not been described in the literature. We note that at such altitudes anisotropic electron-density fluctuations are difficult to observe because of the smallness of their amplitude (the small factor $\alpha_i a_e \bar{N}$).

In (7) a sharp decrease of $E(\rho)$ is predicted in the dissipation interval, approximately according to the law $\exp\{-a(\rho\lambda_0)^2\}$, where a is a constant of order unity. Taking this into account, and also what was stated above, one may arrive at the following conclusion. As the frequency of radio waves is increased in scattering from one and the same region of the ionosphere (or as the height of the scattering region is increased, i.e., as λ_0 is increased, for one and the same radio-wave frequency), isotropic scattering should first be observed, then anisotropic scattering, and with a further increase in frequency—a sharp drop in the intensity of the scattered radiation.

I express my gratitude to A. S. Monin for his attention to this work.

Institute of Atmospheric Physics
Academy of Sciences of the USSR

Received
13 III 1961

References

1. E. A. Novikov, *Izv. AN SSSR, ser. geofiz.*, No. 11 (1960).
2. B. Nichols, *J. Geophys. Res.*, **64**, No. 12 (1959).
3. I. D. Howells, *J. Fluid Mech.*, **8**, part 4 (1960).
4. A. N. Kolmogorov, *DAN*, **30**, No. 4 (1941).
5. J. S. Greenhow, E. L. Neufeld, *J. Geophys. Res.*, **64**, No. 12 (1959).
6. H. G. Booker, *J. Geophys. Res.*, **64**, No. 12 (1959).
7. E. A. Novikov, *DAN*, **139**, No. 2 (1961).
8. G. K. Batchelor, *J. Fluid Mech.*, **5**, part 1 (1959).
9. G. K. Batchelor, J. D. Howells, A. A. Townsend, *J. Fluid Mech.*, **5**, part 1 (1959).

Note: Figure translations are in progress. See original paper for figures.