SovietRxiv

Soviet-era science, translated into English

Reports of the Academy of Sciences of the USSR

A. A. BARMIN

1961

SovietRxiv

View the original and related papers at https://sovietrxiv.org/items/ru-196101.51607

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

SovietRxiv

Abstract

Full Text

Reports of the Academy of Sciences of the USSR

1961. Volume 138, No. 1

HYDROMECHANICS

A. A. BARMIN

DISCONTINUITY SURFACES WITH RELEASE OR ABSORPTION OF ENERGY IN MAGNETIC HYDRODYNAMICS

(Presented by Academician L. I. Sedov on 25 I 1961)

Let us consider discontinuity surfaces with release or absorption of energy in an electrically conducting medium in an arbitrary magnetic field. We shall assume the conductivity of the medium to be infinite, and prescribe the internal energy in the form $p/(\gamma-1)\rho + C$.

The conditions at a discontinuity ([1,2]) can in this case be resolved in the following form:

$$v_1^2 = (\Delta h + h_1)z + 1; \tag{1}$$

$$v_2^2 = h_1 z + 1; \tag{2}$$

$$\Delta P = \Delta h\,[z - \tfrac{1}{2}(\Delta h + 2h_1)]; \tag{3}$$

$$\Delta P^* = \Delta h z; \tag{4}$$

$$\Delta R = \Delta h z/(h_1 z + 1); \tag{5}$$

$$z^2\Delta h[2h_1 - (\gamma-1)\Delta h] - 2z[\Delta h(h_1^2 + \gamma_2 P_1 - 1 + q-$$

$$-\tfrac{1}{2}(\gamma_2 - 2)h_1\Delta h) + h_1 q] - \Delta h[\Delta h + 2h_1] - 2q = 0. \tag{6}$$

SovietRxiv

Index 1 corresponds to quantities ahead of the discontinuity, index 2—behind the discontinuity; $h_i = H_{\tau i}/H_{ni} = \operatorname{tg}\alpha_i$, where α_i is the angle between the magnetic field and the normal to the discontinuity surface ($i = 1, 2$); $-z$ is the cotangent of the angle of rotation of the velocity at the discontinuity; v_1 and v_2 are the velocities of motion of the surface relative to the gas ahead of the jump and behind the jump, divided by the Alfvén velocities $V_{Ai} = H_n/\sqrt{4\pi\rho_i}$; a_+, a_- are respectively the velocities of fast and slow magnetohydrodynamic waves; $\Delta P = P_2 - P_1 =$

$$= \frac{p_2 - p_1}{H_n^2/4\pi}$$

is the jump of dimensionless pressure at the discontinuity; $\Delta R = (\rho_2 - \rho_1)/\rho_1$ is the jump of dimensionless density at the discontinuity;

$$\Delta P^* = \frac{(p_2 + H_2^2/8\pi) - (p_1 + H_1^2/8\pi)}{H_n^2/4\pi}$$

is the jump of total pressure at the discontinuity;

$$q = (\gamma_2 - 1)\frac{Q\rho_1}{H_n^2/4\pi} + \frac{\gamma_2 - \gamma_1}{\gamma_1 - 1}\frac{p_1}{H_n^2/4\pi},$$

where Q is the energy of formation released or absorbed. For exothermic reactions $q > 0$, for endothermic reactions $q < 0$ ([3]).

As in ordinary gas dynamics, we shall consider discontinuities of two kinds: 1) detonation discontinuities, when, with respect to the initial state, first there travels a shock wave, and immediately behind it a reaction zone, and therefore the wave velocity is found from the solution of the problem; 2) discontinuities of the combustion type, when the reaction zone travels through the initial state; the discontinuity velocity is then prescribed.

as an additional relation determined from the purely physicochemical problem.

An investigation of relations (1)–(6) shows that the following types of discontinuities will be evolutionary.

I. Discontinuities with release of energy ($q > 0$)

I. 1A. Super-Alfvén detonation is a discontinuity whose propagation velocities, relative to the gas ahead of and behind the discontinuity, satisfy the inequalities $v_1^2 > a_{1+}^2$, $v_2^2 \leq a_{2+}^2$. In it the reaction zone follows a fast magnetohydrodynamic shock wave. At this discontinuity $\Delta P > 0$, $\Delta P^* > 0$, $\Delta h > 0$, $2/(\gamma_2 - 1) > \Delta R > 0$, and $v_1^2 > v_2^2$. The maximum value of the density jump is attained at $\Delta h = 2h_1/(\gamma_2 - 1)$, independently of the released energy q.

SovietRxiv

I. 1B. Sub-Alfvén detonation is a discontinuity whose propagation velocities satisfy the inequalities $a_{1-}^2 < v_1^2 \leq 1,\ v_2^2 \leq a_{2-}^2$. In this case the reaction zone follows a slow magnetohydrodynamic shock wave. At this discontinuity $\infty > \Delta P > 0,\ \infty > \Delta P^* > 0,\ \infty > \Delta R > 0,\ -h < \Delta h < 0$, and $v_1^2 > v_2^2$. For sufficiently large release of energy $q > q_1$ this type of detonation is absent. Here $q_1 = \frac{1}{2}h_1^2$ for $\delta = \frac{1}{2}\gamma_2 h_1^2 + \gamma_2 P_1 - 1 > 0$; $q_1 = \frac{1}{2}h_1^2 + \delta/2(\gamma_2 + 1)$ for $-1 < \delta < -1 + \sqrt{1 - (\gamma_2^2 - 1)h_1^2}$; the expression for q_1 for $0 > \delta > -1 + \sqrt{1 - (\gamma_2^2 - 1)h_1^2}$ is more cumbersome.

As q increases from 0 to $q_m(v_1)$ at fixed velocity v_1^2, the velocity v_2^2 increases up to the values a_{2+}^2 and a_{2-}^2, respectively, in the super-Alfvén and sub-Alfvén discontinuities, while the further increase of v_2^2 occurs as q decreases. The points at which q is maximal are analogues of the Jouguet points in gas dynamics. As q increases, the jumps of density, total pressure, and magnetic field decrease to certain values corresponding to the Jouguet points.

To calculate a detonation it is sufficient to specify the initial pressures, density, magnetic field, q, and one of the quantities Δh or z, which characterize the intensity of the discontinuity. Then from relation (6) we determine z or Δh, and then from (1)–(5) we find the remaining quantities. We note that, at fixed Δh, the quantities $\Delta P,\ \Delta P^*,\ \Delta R$ for a super-Alfvén detonation are greater than in a fast shock wave, and for a sub-Alfvén detonation they are smaller than in a slow shock wave.

I. 2A. Fast super-Alfvén combustion is a discontinuity whose propagation velocities satisfy the inequalities $v_1^2 > a_{1+}^2,\ v_2^2 \geq a_{2+}^2$. At this discontinuity $\Delta P > 0,\ \Delta P^* > 0,\ \infty > \Delta R > 0,\ \infty > \Delta h > 0$, and $v_1^2 > v_2^2$. This type of combustion corresponds, in gas dynamics, to a flame with forced velocity, sometimes called weak detonation.

I. 2B. Fast sub-Alfvén combustion is a discontinuity whose propagation velocities satisfy the inequalities $a_{1-}^2 < v_1^2 \leq 1,\ a_{2-}^2 \leq v_2^2 \leq 1$. At this discontinuity $\infty > \Delta P \geq 0,\ \infty > \Delta P^* \geq 0,\ \infty > \Delta R \geq 0,\ 0 > \Delta h > -1$, and $v_1^2 \geq v_2^2$. This type of discontinuity does not occur for the same values q_1 as the sub-Alfvén detonation.

As q increases from 0 to $q_m(v_1)$ at constant v_1, the square of the velocity v_2^2 decreases from v_1^2 to the values a_{2+}^2 and a_{2-}^2, respectively, in the fast super-Alfvén and sub-Alfvén discontinuity. For $q = q_m(v_1)$ we have Jouguet points, which are the boundary between detonations and combustion discontinuities of this type. As q increases at constant v_1^2, the jumps of density, total pressure, and field increase from 0 to certain values at the Jouguet points.

I. 2B. **Slow super-Alfvén combustion**—a discontinuity whose propagation velocities satisfy the inequalities $1 \leqslant v_1^2 < a_{1+}^2$, $1 \leqslant v_2^2 \leqslant a_{2+}^2$. At this discontinuity $\Delta P^* < 0$, $\Delta R < 0$, $-1 < \Delta h < 0$, and $v_1^2 \leqslant v_2^2$. The gas-dynamic pressure may either increase or decrease. For sufficiently large energy release ($q > q_2$), this type of discontinuity does not occur; here $2q_2 = h_1^2 + \delta/(\gamma_2 + 1)$ for $\delta \geqslant 0$;

SovietRxiv

$2q_2 = h_1^2$ for $\delta < 0$. This type of discontinuity corresponds in gas dynamics to the combustion regime with subsonic velocity.

I. 2G. **Slow sub-Alfvén combustion**—a discontinuity whose propagation velocities satisfy the inequalities $v_1^2 < a_{1-}^2$; $v_2^2 \leqslant a_{2-}^2$. At this discontinuity $\infty < \Delta P < 0$; $\infty < \Delta P^* < 0$, $\Delta R < 0$, $\Delta h > 0$, and $v_1^2 < v_2^2$. This type of combustion has no counterpart in gas dynamics.

In the cases of slow sub-Alfvén and super-Alfvén discontinuities, as q increases from 0 to $q_m(v_1)$ at constant v_1, the velocity v_2^2 will increase from v_1^2 to a_{2-}^2 and a_{2+}^2, respectively, with a further increase of v_2^2 occurring as q decreases. The point at which $q = q_m$ is the analogue of the Jouguet point in gas dynamics. As q increases, the absolute magnitude of the jumps in total pressure and magnetic-field density increases to certain finite values.

To calculate combustion-type discontinuities, it is sufficient to specify the initial pressure, density, magnetic field, released energy q, and discontinuity velocity v_1. Then from (1) and (6) one determines z and Δh, and from (2)–(5) the remaining quantities.

II. Discontinuities with absorption of energy

II. 1A. **Super-Alfvén discontinuity of absorption of detonation type**—a discontinuity whose propagation velocities satisfy the inequalities $v_1^2 \geqslant a_{1+}^2$, $v_2^2 < a_{2+}^2$. In it the reaction zone follows a fast magnetohydrodynamic shock wave. At this discontinuity $\Delta P^* > 0$, $\Delta R > 0$, $\Delta h > 0$, and $v_1^2 > v_2^2$. The pressure jump is positive for small $|q|$ and negative for large $|q|$. For some $q = q^*(\Delta h)$, $\Delta P = -P_1$, which corresponds to zero pressure behind the discontinuity. At fixed v_1, as q decreases the jumps in density, total pressure, and magnetic field increase. This discontinuity corresponds to the analogous discontinuity in gas dynamics.

III. 1B. **Sub-Alfvén discontinuity of absorption of detonation type**—a discontinuity whose propagation velocities satisfy the inequalities $a_{1-}^2 \leqslant v_1^2 \leqslant 1$, $v_2^2 < a_{2-}^2$. In it the reaction zone goes behind a slow magnetohydrodynamic shock wave. At this discontinuity $1 + 1/h_1^2 \geqslant \Delta P > 0$, $1 \geqslant \Delta P^* > 0$, $\Delta R > 0$, $-1 \leqslant \Delta h \leqslant 0$, and $v_1^2 > v_2^2$. For sufficiently large absorption of energy, $2q < -[2\gamma_2 P + (\gamma_2 - 1)(h_1^2 + 1)]$, this type of discontinuity does not occur. As q decreases from 0 to q_0 at fixed v_1, the density jump increases to infinity, and the jumps in total and gas-dynamic pressure increase to v_1^2, $v_1^2(v_1^2 h_1^2 + h_1^2 + 1)$, respectively, while v_2^2 falls to zero.

IV. 2A. **Fast super-Alfvén discontinuity with endothermic reaction**—a discontinuity whose propagation velocities satisfy the inequalities $v_1^2 \geqslant a_{1+}^2$, $v_2^2 > a_2^2$. At this discontinuity $\Delta P^* < 0$, $\Delta P < 0$, $-1/h_1 < \Delta R < 0$, $-1 < \Delta h < 0$, and $v_1^2 < v_2^2$.

SovietRxiv

V. 2B. **Fast sub-Alfvén discontinuity with endothermic reaction**—a discontinuity whose propagation velocities of propagation ···

···whose propagation velocities satisfy the inequalities $a_{1-}^2 \leqslant v_1^2 \leqslant 1,\ a_{1-}^2 \leqslant v_2^2 \leqslant 1$. At this discontinuity $\Delta P < 0,\ \Delta P^* < 0,\ \Delta R < 0,\ \Delta h > 0$, and $v_1^2 \leqslant v_2^2$.

In both of the last cases, for a given v_1, the absolute magnitude of the jumps in pressure, total pressure, density, and magnetic field increases as q decreases; moreover, for some $q = q^*(v_1)$ the pressure behind the discontinuity becomes negative ($\Delta P < P_1$). In this case, in the super-Alfvénic discontinuity, for

$$q < -\frac{\gamma_2 - 1}{\gamma_2} P_1$$

$\Delta P < -P_1$ for any v_1^2.

II. 2B. **Slow super-Alfvénic discontinuity with endothermic reaction** —a discontinuity whose propagation velocities satisfy the inequalities $1 \leqslant v_1^2 \leqslant a_{1+}^2,\ 1 \leqslant v_2^2 < a_{2+}^2$. At this discontinuity $\Delta P^* > 0,\ \Delta R > 0,\ \Delta h > 0$, and $v_2^2 \leqslant v_1^2$. The jump in ordinary pressure may be either positive or negative.

For a given v_1^2, as q decreases, the jumps in total pressure, density, and magnetic field increase. For $q = q(h_1^2, P_1, v_1^2)$ when

$$v_1^2 > 1 + h_1^2$$

and for $q = 0$ when $v_1 \leqslant 1 + h_1^2$, the pressure jump is zero. With a further decrease in q, ΔP decreases and, at $q = q^*(v_1)$, $\Delta P = -P_1$, which corresponds to zero pressure behind the discontinuity.

II. 2 . **Slow sub-Alfvénic discontinuity with endothermic reaction**— a discontinuity whose propagation velocities satisfy the inequalities $v_1^2 \leqslant a_{1-}^2,\ v_2^2 < a_{2-}^2$. At this discontinuity $\infty > \Delta P > 0,\ \infty > \Delta P^* > 0,\ \Delta R > 0,\ \Delta h < 0$, and $v_1^2 \geqslant v_2^2$. For a given v_1^2, as q decreases, the density jump increases to infinity, while the jumps in pressure, total pressure, and magnetic field increase to certain values; in this process v_2^2 falls to zero. For large energy absorptions,

$$2|q| > 2\gamma_2 P_1 + \frac{(\gamma_2 - 1)h_1^2 + 1}{2}\left(h_1^2 + \gamma_2 P_1 - 1 - \sqrt{(h_1^2 + \gamma_2 P_1 - 1)^2 + 4h_1^2}\right)$$

this type of discontinuity does not occur.

There is no counterpart of the sub-Alfvénic types of discontinuities in ordinary gas dynamics. Discontinuities with absorption of energy are calculated in the same way as in the cases with release of energy.

SovietRxiv

Research Institute of Mechanics
Moscow State University
named after M. V. Lomonosov

Received
24 I 1961

CITED LITERATURE

1. L. D. Landau, E. M. Lifshitz, *Mechanics of Continuous Media*, 1953, pp. 569–598.

2. L. D. Landau, E. M. Lifshitz, *Electrodynamics of Continuous Media*, 1957, p. 290.

3. G. Courant, K. Friedrichs, *Supersonic Flow and Shock Waves*, IL, 1950, p. 199.

4. I. A. Akhiezer, G. Ya. Lyubarskii, R. V. Polovin, *ZhETF*, **35**, 731 (1958).

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.