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Abstract

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THE RECIPROCITY LAW AND THE QUASI-STATIONARITY OF ELECTRONIC PROCESSES IN PHOTOELECTRETS

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1. The reciprocity law (r.l.) for photoelectrets may be formulated as the assertion that the volume charge that is formed (during polarization) or that flows off (during depolarization) as a result of illumination of a crystal in an electric field depends only on the exposure $z = It$, and not on the light intensity I and the illumination time t separately:

$$Q = \int \rho(x,t)\, dx = Q(z). \tag{1}$$

Here the integral is extended over the near-surface layer in which the volume-charge density $\rho \neq 0$. A number of works ([1–9]) have been devoted to the study of the r.l. in electrophotography; however, the theoretical question of the conditions for fulfillment of the r.l. in photoelectrets remains unresolved.

The necessary conditions were obtained in the form of the following inequalities ([3,4]):

$$n \ll |P - N|; \qquad \frac{\partial n}{\partial t} \ll \left| \frac{\partial P}{\partial t} - \frac{\partial N}{\partial t} \right| \qquad (0 \le x \le l). \tag{2}$$

The following notation is used: n, N, P, M_1, M_2 are, respectively, the concentration of conduction electrons, electrons at trapping levels, holes at activator levels, activator centers, and trapping centers (see Fig. 1); l is the thickness of the crystal in the direction x, in which the field is applied and illumination is carried out; $\varkappa$ is the dielectric constant; ε_i is the binding energy of an electron on a local level of the i-th type.

Necessary and sufficient conditions were determined only for the case of large exposure times, weak excitation of donors, and weak filling of acceptors ([10]). It turned out that they are expressed by the inequality

$$n(x,t) \ll N(x,t) \qquad (0 \le x \le l). \tag{3}$$

Relations (2) and (3) are different forms of the conditions for quasi-stationarity of the concentration of mobile charge carriers ([11]). In the present article a general consideration is given of the question of the necessary and sufficient conditions for fulfillment of the r.l. in photoelectrets and of the connection between the r.l. and the quasi-stationarity of the kinetics of electronic processes.

2. Let us consider a crystal for which the energy spectrum and the scheme of electronic transitions are shown in Fig. 1. The kinetics of electronic processes under illumination of the crystal in an external field E_1 is described by the following system of equations:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial x} = 0; \qquad & \frac{\partial N}{\partial t} = -k_2 N + \beta n (M_2 - N); \\ \frac{\partial E}{\partial x} = \frac{4\pi}{\varkappa}\rho; \qquad & \frac{\partial P}{\partial t} = k_1 (M_1 - P) - \alpha n P; \\ q\mu n E + qD\frac{\partial n}{\partial x} = j; \qquad & \frac{1}{q}\rho = P - N - n, \end{aligned} \tag{4}$$

where $k_1 = k_1(T, I)$; $k_2 = k_2(T, I)$.

The corresponding boundary conditions have the form

$$j(0,t) = 0; \qquad E(0,t) = E_1/\varkappa; \qquad j(l,t) = 0; \qquad E(l,t) = E_1/\varkappa. \tag{5}$$

The case of formation of a photoelectret (polarization) corresponds to zero initial conditions

$$n(x,0) = N(x,0) = P(x,0) = j(x,0) = \rho(x,0) = E(x,0) - E_1/\varkappa = 0. \tag{6}$$

In depolarization, the initial conditions are given by the stationary solution of the problem of the distribution of concentrations, charge, and field in the photoelectret [(10)].

Let us pass from the variables x, t to the variables x, z ($z = It$). Without loss of generality the functions n and j, as well as the transition probabilities k_1 and k_2, may be represented in the following form:

$$n = I n_g; \qquad j = I j_g; \qquad k_1 = s_1 I; \qquad k_2 = s_2 I. \tag{7}$$

Here n_g and j_g are new unknown functions of x, z, and, possibly, of the parameter I; in the general case the coefficients s_1 and s_2 may depend on temperature and on the intensity of light.

After the introduction of the new variables, the system of equations (4) assumes the form

Fig. 1

Figure 1: Fig. 1

$$\frac{\partial \rho}{\partial z} + \frac{\partial j_g}{\partial x} = 0; \qquad \frac{\partial N}{\partial z} = -s_2 N + \beta n_g (M_2 - N);$$
$$\frac{\partial E}{\partial x} = \frac{4\pi}{\varkappa}\rho; \qquad \frac{\partial P}{\partial z} = s_1 (M_1 - P) - \alpha n_{gP}; \tag{8}$$
$$q\mu n_{gE} + qD\frac{\partial n_g}{\partial x} = j_g; \qquad \frac{1}{q}\rho = P - N - In_g.$$

Fig. 1

If: 1) for all types of local levels the rates of thermal generation of free charge carriers are negligibly small in comparison with the rates of optical generation ($s_1 = \text{const}$, $s_2 = \text{const}$), and 2) in the last equation of system (8) the term In_g may be neglected, then the system of equations (8), as well as the boundary and initial conditions, do not contain I. Consequently, under these and only under these conditions the reciprocity law is fulfilled:

$$Q = \int \rho \, dx \simeq q \int [P(x,z) - N(x,z)] \, dx = Q(z). \tag{9}$$

3. Noting that the probabilities of thermal emission of electrons k_1^{therm} and k_2^{therm} are related to the capture probabilities α and β by the formulas [11]

$$k_1^{\text{therm}} = \alpha \cdot 2 \left(\frac{2\pi mkT}{h^2}\right)^{3/2} e^{-\varepsilon_1/kT}, \qquad k_2^{\text{therm}} = \beta \cdot 2 \left(\frac{2\pi mkT}{h^2}\right)^{3/2} e^{-\varepsilon_2/kT}, \tag{10}$$

we arrive at the inequalities

$$I \gg 2 \left(\frac{2\pi mkT}{h^2}\right)^{3/2} e^{-\varepsilon_1/kT} \frac{\alpha}{s_1}, \qquad I \gg 2 \left(\frac{2\pi mkT}{h^2}\right)^{3/2} e^{-\varepsilon_2/kT} \frac{\beta}{s_2}, \tag{11}$$

under which the first of the conditions of the reciprocity law obtained above is realized.

4. From systems (4), (5), and (6) it follows that, for small t, the current j and the concentrations N, P, and n are quantities of first order of smallness, while ρ and $\Delta E = E - E_1/\varkappa$ are quantities of second order of smallness. In the linear approximation the system (4) decomposes into two, with the last three

the equations take the form

$$\frac{\partial N}{\partial t} + k_2 N - \beta M_2 n = 0, \qquad \frac{\partial P}{\partial t} + k_1 P = k_1 M, \qquad P - N - n = 0 \tag{12}$$

and can be solved independently of the remaining ones. For $t \ll 1/k_1$,

$$P \approx k_1 M_1 t,$$

$$N \approx \frac{k_1 M_1 \beta M_2}{\beta M_2 + k_2 - k_1} t - \frac{k_1 M_1 \beta M_2}{(\beta M_2 + k_2)(\beta M_2 + k_2 - k_1)} \left[1 - e^{-(\beta M_2 + k_2)t}\right], \tag{13}$$

$$n \approx \frac{k_1 (k_2 - k_1) M_1}{\beta M_2 + k_2 - k_1} t + \frac{k_1 M_1 \beta M_2}{(\beta M_2 + k_2)(\beta M_2 + k_2 - k_1)} \left[1 - e^{-(\beta M_2 + k_2)t}\right].$$

If $t \ll \dfrac{1}{\beta M_2 + k_2}$, then $n \approx P \gg N$, and therefore neglecting the term In_g in the last equation of system (8) is inadmissible.

For

$$\frac{1}{\beta M_2 + k_2} \ll t \ll \frac{1}{k_1}$$

$$\frac{n}{N} \approx \frac{1}{(\beta M_2 + k_2)t} + \frac{k_2 - k_1}{\beta M_2}. \tag{14}$$

If $k_1 \ll \beta M_2,\ k_2 \ll \beta M_2$, then at $t \sim 10/\beta M_2$ a quasistationary regime $n \ll N$ arises. This inequality is preserved for all $t \geq 10/\beta M_2$, if the characteristic time $\tau = 1/\beta M_2$ is much less not only than $1/k_1$ and $1/k_2$, but also than the Maxwell relaxation time $\tau_M = \varkappa/4\pi q\mu n_{\text{cv}}$, where $n_{\text{cv}} \sim k_1 M_1/\beta M_2$. Consequently, if the inequalities

$$I \ll \frac{\beta M_2}{s_1}, \qquad I \ll \frac{\beta M_2}{s_2}, \qquad I \ll \frac{\varkappa(\beta M_2)^2}{4\pi q\mu s_1 M_1}, \tag{15}$$

are satisfied, then for $t \gg \tau$ one may neglect n and set $\rho = q(P-N)$ everywhere, except for the quasineutral region in the interior of the crystal, where $P \approx N$. Since in this region $\rho \approx 0$, it makes no noticeable contribution to the space charge.

Let us now choose the state of the crystal at $t = \tilde{t} = 10\tau$ as the initial conditions of the system of kinetic equations (8). For $t \geq \tilde{t}$, the intensity I enters this system of equations only through the exposure z. Since the boundary conditions (5) likewise do not depend on I, deviations from the law of reciprocity can be caused only by the initial conditions, i.e., by the values of N, P, n_g, ρ, j_g, and ΔE at $t = \tilde{t}$, if these values depend explicitly on I. However, according to (13), the quantities

$$N(x,\tilde{t}) = s_1 M_1 \tilde{z}, \qquad P(x,\tilde{t}) \approx s_1 M_1 \tilde{z}, \qquad n_g(x,\tilde{t}) \approx \frac{s_1}{\beta M_2}, \qquad j_g(x,\tilde{t}) \approx \frac{q\mu s_1 E_1}{\beta M_2 \varkappa},$$

$$\Delta E(x,\tilde{t}) \approx 0, \qquad \rho(x,\tilde{t}) \approx 0, \tag{16}$$

do not depend on I for the given value $\tilde{z} = I\tilde{t}$.

The formation of space charge and the appearance of a coordinate dependence of the solution are caused in a photoelectret by effects near the surfaces $x = 0$ and $x = l$. The characteristic time for these effects is the Maxwell relaxation time $\tau_M = \varkappa/4\pi q\mu n_{\text{cv}}$, where $n_{\text{cv}} = k_1 M_1/\beta M_2$. For $\tau_M/\tau \gg 1$, the distortions of the initial conditions by near-surface effects are small*. Therefore one may neglect the contribution of these effects to the solution and assume that at the time $\tilde{t}$ the initial conditions (16), which do not explicitly depend on I, are preserved. But in this case, for all $t > \tilde{t}$ the dependence on I will enter the solution only through the exposure $z = It$, i.e., the law of reciprocity will be satisfied.

* An estimate of the majorant expressions $\rho = qP(t)$ and $Q = qP(t)\mu E_1 t/\varkappa$ at $t = \tilde{t}$ shows that $\rho \ll 10^{-11}$ C/cm^3, $Q \ll 10^{-14}$ C/cm^2.

Thus it has been proved that the necessary and sufficient conditions for fulfillment of the law of reciprocity in the polarization of a photoelectret are expressed by inequalities (11) and (15), which relate the light intensity to the microparameters of the crystal.

It is proved analogously that the same conditions for fulfillment of the law of reciprocity are valid for the process of depolarization of a photoelectret under illumination.

5. Since, when and only when inequalities (15) are fulfilled, a quasistationary concentration of mobile charge carriers is created in the crystal, another equivalent mathematical expression of the necessary and sufficient conditions for the law of reciprocity in photoelectrets is given by inequalities (11) and the quasistationarity conditions in the form

$$n(x,t) \ll N(x,t), \qquad \frac{\partial n(x,t)}{\partial t} \ll \frac{\partial N(x,t)}{\partial t} \left(0 \leq x \leq l;\ t > \frac{10}{\beta M_2}\right). \tag{17}$$

6. From the discussion given above it follows that, when the law of reciprocity is fulfilled, the following dependence of the current density on the light intensity must hold:

$$j(x,t) = j_g(x,z)\,I. \tag{18}$$

Therefore, verification of relation (18) can serve as a method for indicating deviations from the law of reciprocity; if the experimentally determined ratio j/I depends on I at a given z, this unambiguously indicates a violation of the law of reciprocity.

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Note: Figure translations are in progress. See original paper for figures.