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# HYDROMECHANICS

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**Abstract**

**Full Text**

*HYDROMECHANICS*

**V. E. NEUVAZHAEV**

## OUTFLOW OF GAS INTO A VACUUM UNDER A POWER-LAW ENERGY-RELEASE LAW

*(Presented by Academician A. D. Sakharov on 30 VI 1961)*

This note considers the problem of the outflow of gas into a vacuum under a power-law law of energy release. The problem is self-similar and differs in that it reduces to solving a system of two ordinary differential equations, rather than one, as is usual in problems of one-dimensional motion. The required integral curve passes through two non-isolated singular points. From each point there issues an infinite set of integral curves. In numerical integration from point to point, rounding errors grow exponentially; therefore the usual methods of numerical integration are not suitable here. The solution is found by an iterative method, after suitable variables have been chosen.

**1. Formulation of the problem\***. There is a half-space filled with an ideal gas which at the initial moment  $t = 0$  is at rest; the density is constant,  $\rho = \rho_0$ , and the pressure  $p_n = 0$ . Per unit mass, according to the power law  $\varepsilon_0 = At^n$ ,  $A > 0$ ,  $n > 0$ , energy release is prescribed. The gas borders on a vacuum; therefore for  $t > 0$  a rarefaction wave will propagate through the quiescent gas with sound speed  $c_0(t) = \sqrt{\gamma(\gamma - 1)At^n}$ ;  $\gamma$  is the adiabatic exponent. The free boundary ( $p = 0$ ) will fly into the vacuum. The motion is described by the equations of gas dynamics. In Lagrangian coordinates they have the form

$$\frac{\partial(1/\rho)}{\partial t} - \frac{\partial u}{\partial m} = 0, \quad \frac{\partial u}{\partial t} + \frac{\partial p}{\partial m} = 0, \quad \frac{\partial \varepsilon}{\partial t} + p \frac{\partial(1/\rho)}{\partial t} = \frac{\partial \varepsilon_0}{\partial t}, \quad (1)$$

where  $t$  is time;  $m$  is the mass of gas, measured from the boundary at the moment  $t = 0$ ;  $\rho$  is density;  $p$  is pressure;  $u$  is velocity;  $\varepsilon$  is specific internal energy.

The solution is sought in the form

$$p = p_0(t)\pi(\alpha), \quad \rho = \rho_0\delta(\alpha), \quad u = c_0(t)\xi(\alpha), \quad (2)$$

where  $\alpha = \frac{m}{M(t)}$  is the self-similar independent variable;  $M(t) =$

$\frac{2}{n+2} \sqrt{\gamma(\gamma-1)} A t^{1+n/2} \rho_0$  is the mass of the disturbed gas;  $p_0(t) = (\gamma-1) A t^n \rho_0$  is the pressure of the quiescent gas. After substituting (2) into (1), we obtain the system of ordinary differential equations

$$\delta' = \frac{N\delta^2[\gamma\alpha\xi + 2(\pi - \delta)]}{\gamma\alpha(\alpha^2 - \pi\delta)}, \quad \xi' = \frac{N[\gamma\alpha\xi + 2(\pi - \delta)]}{\gamma(\alpha^2 - \pi\delta)}, \quad (3)$$

$$\pi' = N \frac{\gamma\xi\pi\delta + 2\alpha(\pi - \delta)}{\alpha^2 - \pi\delta}, \quad N = \frac{n}{n+2}.$$

\* The problem was posed by E. I. Zababakhin. The asymptotics (5) were indicated by L. P. Feoktistov.

and the boundary conditions

$$\alpha = 0 : \quad \pi = 0; \quad \alpha = 1 : \quad \pi = 1, \quad \zeta = 0, \quad \delta = 1. \quad (4)$$

**Item 2.** It is known that the adiabatic outflow of a gas into a vacuum occurs with a finite velocity, while the isothermal outflow occurs with an infinite one. Under a power-law energy-release law the gas flows out with infinite velocity. This is proved by contradiction when considering system (3) in a neighborhood of the point  $(\alpha = 0, \pi = 0, \zeta_0, \delta_0)$ . We shall denote by the subscript 0 the values of  $\pi, \delta, \zeta$  at  $\alpha = 0$ .

**Fig. 1**

**Fig. 2**

**Item 3.** The required solution must satisfy system (3) and the boundary conditions (4). The points  $(\alpha = 0, \pi = 0, \delta_0, \zeta = -\infty)$  and  $(\alpha = 1, \pi = 1, \delta = 1, \zeta = 0)$  are nonisolated singular points of system (3). For a numerical solution it is important to know the behavior of the integral curves in a neighborhood of the indicated points. If one introduces the new variables  $\Pi = \pi/\alpha, \Delta = \delta/\alpha$ , then the problem reduces to a system of two equations for  $\Pi$  and  $\Delta$  with respect to the independent variable  $\zeta$ . In the new variables it can be shown that, for  $\zeta = -\infty, \Pi = \infty, \Delta = \infty$ , and moreover:

$$\frac{\zeta_0}{\Pi_0} = -\frac{1}{N\gamma}, \quad \frac{\zeta_0}{\Delta_0} = -\frac{2}{\gamma(\gamma-1+2N)}. \quad (5)$$

Using (5), one can investigate in greater detail the point I ( $\zeta = -\infty, \Pi = \infty, \Delta = \infty$ ) and show that an infinite set of integral curves having the same slope passes through it. Through the point II ( $\zeta = 0, \Pi = 1, \Delta = 1$ ) there also passes an infinite set of integral curves having the same slope.

**Fig. 3**

**Item 4. Method of solution.** In numerical integration from point I to point II, rounding errors grow exponentially. The same occurs when integrating from point II to point I.

For the numerical solution of the problem the following iteration method is used. Such combinations of the functions  $\Pi$  and  $\Delta$  are found, one of which

is counted stably from point I to point II, the other from point II to point I. It turns out that these functions will be  $\theta = \Pi/\Delta$  and  $x = 1/\Pi$ . The function  $x$ , for a prescribed  $\theta$ , is counted stably from point I to point II; the function  $\theta$ , for a prescribed  $x$ , from point II to point I. To leave the singular point I, we use the expansion for  $x$ :

$$x = -\frac{1}{N\gamma\xi} + \frac{N+1}{N^2\gamma} \frac{1}{\xi} \exp\left[\frac{\gamma}{4}(\gamma-1+2N)\xi^2\right] \int^{\xi} \exp\left[\frac{\gamma}{4}(\gamma-1+2N)\zeta^2\right] \frac{d\zeta}{\zeta} + \dots \quad (6)$$

From point II we leave by means of the expansion for  $\theta$ :

$$\theta = 1 + (\gamma-1)\xi + \dots \quad (7)$$

Introduce a new independent variable  $z$ :  $z = \zeta/(1-\zeta)$ . System (3) in the variables  $z, \theta, x$  has the form:

$$\begin{aligned} \frac{d\theta}{dz} &= \frac{\gamma(\gamma-1)x\theta z + 2(\theta-1)(1+z)(\gamma\theta x^2 - 1)}{[\gamma\theta x z + 2(\theta-1)(1+z)]x(1+z)^2}, \\ \frac{dx}{dz} &= \frac{\frac{\gamma}{N}(x^2\theta - 1)(1+z) - \gamma^2 x z - 2\gamma(\theta-1)x^2(1+z)}{[\gamma\theta x z + 2(\theta-1)(1+z)](1+z)^2}. \end{aligned} \quad (8)$$

Let  $z_0$  and  $z_\omega$  be points of the interval  $(-1, 0)$ , taken respectively in the neighborhoods of the points  $z = -1$  and  $z = 0$ . The interval  $(z_0, z_\omega)$  is divided by points into  $\omega$  subintervals. At the division points the function  $\theta_i^0$ ,  $i = 0, 1, \dots, \omega$ , is prescribed arbitrarily. Using expansion (6), we numerically integrate the second equation of system (8). From expansion (7) we compute  $\theta_\omega^1$ , and then from the first equation (8) we find all  $\theta_i^1$ ,  $i = \omega-1, \dots, 0$ . The  $\theta_i^1$  found are used to recompute  $x_i^0$ , and the  $x_i^1$  to recompute  $\theta_i^1$ . The iterations are continued until the conditions

$$|\theta_i^{\nu+1} - \theta_i^\nu| < \varepsilon, \quad |x_i^{\nu+1} - x_i^\nu| < \varepsilon,$$

$$i = 0, 1, \dots, \omega.$$

After the solution has been obtained in the variables  $z, x, \theta$ , we pass to the variables  $\alpha, \pi, \delta, \zeta$ :

$$\delta(z) = \exp\left(\int_0^z \frac{dz}{\theta x(1+z)^2}\right), \quad \alpha(z) = \theta(z)x(z)\delta(z), \quad \pi(z) = \theta(z)\delta(z), \quad \zeta(z) = \frac{z}{1+z}. \quad (9)$$

5. The gas flow is described by the following model: through a stationary background with pressure  $p_0(t)$  and density  $\rho_0$ , a rarefaction wave propagates with velocity  $c_0(t)$ ; the outflow velocity is infinite. For fixed  $t$ , the pressure, density, and velocity vary monotonically with  $m$ :

$$0 \leq p \leq p_0(t), \quad 0 \leq \rho \leq \rho_0, \quad -\infty \leq u \leq 0.$$

If  $m = 0$ , then

$$\frac{\partial p}{\partial m} = \frac{\partial \rho}{\partial m} = \frac{\partial u}{\partial m} = \infty;$$

if  $m = M(t)$ , then

$$\frac{\partial p}{\partial m} = \frac{p_0(t)}{M(t)}\gamma a, \quad \frac{\partial \rho}{\partial m} = \frac{\rho_0}{M(t)}a, \quad \frac{\partial u}{\partial m} = \frac{c_0|t|}{M(t)}a, \quad a = \frac{2 - (3 - 2/\gamma)N}{1 + \gamma}$$

for

$$\frac{2}{N(\xi - 2/\gamma)} > 1$$

and

$$\frac{\partial p}{\partial m} = \frac{\partial \rho}{\partial m} = \frac{\partial u}{\partial m} = 0$$

for

$$\frac{2}{N(\xi - 2/\gamma)} \leq 1.$$

Figures 1, 2, and 3 give the results of calculations: curves 1 correspond to  $N = 0.5$ ,  $\gamma = 1.4$ , curves 2 to  $N = 0.9$ ,  $\gamma = 3$ .

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*Note: Figure translations are in progress. See original paper for figures.*

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