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Abstract

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On the Question of the Fermion Mass in a γ^5-Invariant Model of Quantum Field Theory

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In a number of recently published works (^{1}) an attempt has been made to obtain a finite fermion mass in γ^5-invariant theories. The mass corrections calculated in any order of perturbation theory are, in this case, equal to zero. This circumstance, however, does not prove that a mass cannot arise as a result of the interaction, since for the correct application of perturbation theory, according to the proposition put forward by N. N. Bogolyubov (^{2}), the degeneracy must first be removed. For example, the removal of the degeneracy associated with the conservation of the number of particles makes it possible to obtain the correct solution in the theory of superconductivity, which cannot be obtained by ordinary perturbation theory. In considering the question of the fermion mass, the degeneracy is associated with invariance with respect to the group of γ^5-transformations:

$$\psi \to e^{\alpha\gamma^5}\psi, \qquad \overline{\psi} \to \overline{\psi}e^{\alpha\gamma^5}. \tag{1}$$

After the degeneracy has been removed, quantities that are not invariant with respect to γ^5-transformations may acquire finite increments; in particular, the appearance of a fermion mass is possible.

In contrast to works (^{1}), we shall consider models in which there are no divergences, namely systems of fermion fields interacting with a real vector field in two-dimensional space-time. The model of the interaction of a massless fermion with a vector meson possessing mass was studied in works (^{3}), where it was shown that, by means of a canonical transformation, it reduces to a problem without interaction and, consequently, the Green' s function has no additional poles except $p^2 = 0$. We shall apply the proposed method first to this model, since here we have the possibility of comparing the result with the exact solution.

The Lagrangian of the system under consideration has the form (we shall follow the notation of (^{4})):

$$\mathcal{L}(x) = \mathcal{L}_0(x) + \mathcal{L}_I(x),$$

$$\mathcal{L}_0(x) = \frac{i}{2}\sum_n \left\{ :\overline{\psi}(x)\gamma^n \frac{\partial\psi}{\partial x^n}: - :\frac{\partial\overline{\psi}}{\partial x^n}\gamma^n\psi(x): \right\} -$$

$$-\frac{i}{2}\sum_{k,n} g^{kk}g^{nn} : \frac{\partial A_k}{\partial x^n}\frac{\partial A_k}{\partial x^n} : + \frac{\mu^2}{2}\sum_n g^{nn} : A_n(x)A_n(x):, \tag{2}$$

$$\mathcal{L}_I(x) = g\sum_n :\overline{\psi}(x)\gamma^n\psi(x)A_n(x):, \qquad n,k = 0,1.$$

Here ψ is the operator of the fermion field, and A_n are the operators of a real vector field with commutation in diagonal form.

In accordance with the preceding remarks, let us introduce into the Lagrangian an infinitely small term

$$-\lambda : \bar{\psi}(x)\psi(x):,$$

which removes the degeneracy connected with the group (1). Assuming the possibility of the appearance of a mass, we write the complete Lagrangian in the form

$$\begin{aligned} \mathcal{L}(x) &= \mathcal{L}'_0(x) + \mathcal{L}'_I(x), \\ \mathcal{L}'_0(x) &= \mathcal{L}_0(x) - m : \bar{\psi}(x)\psi(x):, \\ \mathcal{L}'_I(x) &= \mathcal{L}_I(x) + (m-\lambda) : \bar{\psi}(x)\psi(x): . \end{aligned} \tag{3}$$

Since the fermion mass has been explicitly taken into account in the redefined free Lagrangian $\mathcal{L}'_0$, we shall require that all mass corrections in the sum be equal to zero. This requirement leads to the equation

$$\Sigma(p)\big|_{p^2=m^2} = \lambda - m + \Sigma^*(p)\big|_{p^2=m^2} = 0, \tag{4}$$

where $\Sigma(p)$ is the complete mass operator obtained from the interaction Lagrangian $\mathcal{L}'_I$. We shall call equation (4) the “compensation equation,” by analogy with the theory of superconductivity (5). Using (1) and the explicit form of the Lagrangian (3), one can show the invariance of the compensation equation with respect to the group of γ^5-transformations. Equation (4), in the lowest order of perturbation theory under the condition $g^2/\mu^2 \ll 1$, has the form:

$$m - \lambda = \frac{g^2 m}{2\pi\mu^2}\ln\frac{\mu^2}{m^2}. \tag{5}$$

For $\lambda \to 0$, this equation has solutions: the trivial one, which corresponds to perturbation theory, $m = 0$, and a nontrivial one, $m^2 = \mu^2 e^{-2\pi\mu^2/g^2}$, containing a nonanalytic dependence on the coupling constant. In order to obtain higher approximations in equation (4), we shall apply the renormalization-group method $^{(4)}$, taking into account the compensation of mass corrections. The invariant charge in this case turns out to be equal to unity, and the compensation equation takes the form

$$m \exp\left\{-\frac{g^2}{2\pi\mu^2}\ln\frac{\mu^2}{m^2}\right\} = 0. \tag{6}$$

Equation (6), in contrast to (5), has only the zero solution, which agrees with the conclusions obtained from the exact solution of the model.

Let us apply the method described above to a two-fermion model with vector coupling and with the interaction Lagrangian

$$\mathcal{L}_I = \sum_n : \left\{ g_1\bar{\psi}\gamma^n\psi + g_2\bar{\chi}\gamma^n\chi + \frac{g}{\sqrt{2}}\left(\bar{\chi}\gamma^n\psi + \bar{\psi}\gamma^n\chi\right)\right\} A_n :, \tag{7}$$

where ψ and χ are the operators of two different spinor fields.

Assuming the possibility of the emergence of a mass for both fermions, we transform the Lagrangian analogously to (3) and obtain the system of compensation equations

$$\begin{aligned} \Sigma_1(p)\big|_{p^2=m_1^2} &= \lambda_1 - m_1 + \Sigma_1^*(p)\big|_{p^2=m_1^2} = 0, \\ \Sigma_2(p)\big|_{p^2=m_2^2} &= \lambda_2 - m_2 + \Sigma_2^*(p)\big|_{p^2=m_2^2} = 0, \end{aligned} \tag{8}$$

where $\Sigma_{1,2}(p)$ are the complete mass operators of the fields ψ and χ.

Let us consider the system of compensation equations in the lowest order of perturbation theory for $g_1^2/\mu^2,\ g_2^2/\mu^2,\ g^2/\mu^2 \ll 1$:

$$\begin{aligned} m_1 - \lambda_1 &= \frac{g_1^2 m_1}{2\pi\mu^2}\ln\frac{\mu^2}{m_1^2} + \frac{g^2 m_2}{2\pi\mu^2}\ln\frac{\mu^2}{m_2^2}, \\ m_2 - \lambda_2 &= \frac{g_2^2 m_2}{2\pi\mu^2}\ln\frac{\mu^2}{m_2^2} + \frac{g^2 m_1}{2\pi\mu^2}\ln\frac{\mu^2}{m_1^2}. \end{aligned} \tag{9}$$

For $\lambda_{1,2} = 0$, besides the trivial solution $m_1 = m_2 = 0$, there is also a nontrivial one, which to logarithmic accuracy can be written in the form

$$m_1^2 - m_2^2 \sim m^2 = \mu^2 \exp\left\{-\frac{\pi\mu^2}{g_1^2 g_2^2 - g^4}\left(g_2^2 + g_2^2 - \sqrt{(g_1^2 - g_2^2)^2 + 4g^4}\right)\right\},$$

$$\frac{m_1^2}{m_2^2} = \frac{g_1^2 - g_2^2 + \sqrt{(g_1^2 - g_2^2)^2 + 4g^4}}{g_2^2 - g_1^2 + \sqrt{(g_1^2 - g_2^2)^2 + 4g^4}}. \tag{10}$$

It is seen from (10) that the exponent is negative; consequently, $m^2 \ll \mu^2$, and the solution has a "superconducting character." Here, as in the preceding problem, one can sum certain classes of principal diagrams; in doing so, the result obtained in the lowest order is not changed qualitatively. Of course, the question remains open as to the correctness of summing the selected classes of diagrams and, consequently, as to the possibility of a superconducting solution for the mass. In the theory of superconductivity N. N. Bogolyubov[6] showed that, for the model Hamiltonian of Bardeen, the solution of the compensation equation asymptotically coincides with the exact solution. This also gives us grounds to hope that the solution of the compensation equation qualitatively reflects the features of the exact solution.

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Note: Figure translations are in progress. See original paper for figures.