GEOPHYSICS

1961

SovietRxiv

Abstract

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GEOPHYSICS

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SOME HYDRODYNAMIC EFFECTS IN NON-STATIONARY PURELY WIND-DRIVEN CURRENTS

(Presented by Academician V. V. Shuleikin, 14 IX 1960)

We investigate the process of formation of a surface current under the action of a wind that has arisen suddenly and then changes with time according to some law, in two limiting cases: a strongly stratified and a homogeneous deep sea. The first case corresponds to the occurrence, at a depth h, of so strong a layer of discontinuity in the density of water that the Archimedean forces prove sufficient for the complete damping of turbulence in it. As a result, the transfer of wind energy to the lower layers of water by means of the vertical turbulent flux of momentum ceases, and, at least at the lower boundary of the density-jump layer, slip conditions are practically realized. The second case is readily obtained for $h \to \infty$, since in this case the modulus of the current velocity automatically tends to zero.

Wishing to remain basically within the framework of the classical Ekman scheme, we write the equations of motion for the complex velocity $w = u + iv$ in the form*

$$\frac{\partial w}{\partial t} + i\alpha w = \nu \frac{\partial^2 w}{\partial z^2} \tag{1}$$

with boundary conditions

$$\left.\frac{\partial w}{\partial z}\right|_{z=0} = \tau_1(t), \qquad \tau_1 = \frac{\tau(t)}{\rho\nu}, \qquad \left.\frac{\partial w}{\partial z}\right|_{z=h} = 0, \tag{2}$$

where α is the Coriolis parameter; ν is the coefficient of the kinematic viscosity of water; $\tau(t)$ is the complex tangential wind stress at the surface of the water; z is the vertical coordinate directed downward.

Applying the Laplace integral transform to (1) and (2), and taking into account that, by condition, $w(0, z) = 0$, we obtain

$$\frac{\partial^2 w'}{\partial z^2} = a_1^2 w' \quad \text{for} \quad \left.\frac{\partial w'}{\partial z}\right|_{z=0} = \tau_1'(s) \quad \text{and} \quad \left.\frac{\partial w'}{\partial z}\right|_{z=h} = 0, \tag{3}$$

where

$$w'(z,s) = \int_0^\infty w(z,t) e^{-st}\, dt, \qquad a_1^2 = \frac{s + i\alpha}{\nu}.$$

The solution of (3) in images and its original will be, respectively:

$$w'(z,s) = \frac{\tau_1'}{a_1} \frac{\operatorname{ch} a_1(h-z)}{\operatorname{sh} a_1 h}, \qquad w(zt) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \frac{\tau_1'}{a_1} \frac{\operatorname{ch} a_1(h-z)}{\operatorname{sh} a_1 h} e^{st}\, ds.$$

All poles of the function standing under the sign of the contour integral are simple. Applying Jordan' s lemma and using Borel' s theorem, we finally obtain

$$w(z,t) = \frac{2\nu}{h} \sum_{n=1}^{\infty} (-1)^n \cos n\pi \left(-1\frac{z}{h}\right) \int_0^t \tau_1(t-\xi) e^{-a_n \xi}\, d\xi, \qquad t > 0. \tag{4}$$

* In the Northern Hemisphere of the Earth.

This is the general solution of the problem that we seek. In the general case, however, it is impossible to analyze it. Therefore it is meaningful to represent the arbitrary function $\tau_1(t)$ in the form of a series

$$\tau_1(t) = \sum_{k=-\infty}^{\infty} C_k e^{i\sigma_k t},$$

where

$$C_k = \frac{1}{T} \int_0^T \tau_1(\xi) e^{-i\sigma_k \xi}\, d\xi;$$

$[0, T]$ is the segment on which the variation of the wind is considered; σ_k is the angular frequency of the harmonic.

Then, summing the series in (4) with respect to the number n, we obtain:

$$w(z,t) = \frac{1-i}{2\rho\nu} \sum_{k=-\infty}^{\infty} \frac{\operatorname{ch} a_2(1+i)(h-z)}{a_2 \operatorname{sh} a_2(1+i)h} C_k e^{i\sigma_k t} -$$

Fig. 1

Figure 1: Fig. 1

$$-\frac{1}{\rho h}\sum_{k=-\infty}^{\infty} C_k \sum_{n=1}^{\infty} \frac{e^{-a_n t}}{a_n + i\sigma_k} \cos n\pi \frac{z}{h}, \qquad t > 0, \tag{5}$$

where $a_n = i\alpha + \mu_n^2 \nu,\ \mu_n = \pi n/h,\ a_2^2 = (\alpha + \sigma_k)/2\nu$.

The last result is, naturally, only another form of writing (4) (with certain quite general restrictions imposed on $\tau_1(t)$ when it is expanded in a Fourier series). However, we are now already able to carry out a sufficiently detailed analysis of the general solution obtained above. It is evident, in particular, that the solution has split into two parts: the first sum in (5) describes the flow established in accordance with the given (time-varying) wind, while the double sum describes the result of the influence on the flow, at the first stages of its development, of the forces of inertia and viscosity. The influence of these forces is manifested in the occurrence of damped inertial oscillations, whose frequency is determined only by the Coriolis parameter (and, consequently, depends only on the latitude of the place), while the degree of damping depends on the depth of occurrence of the discontinuity layer and on the viscosity of the water.

Fig. 1

At the same time, for sufficiently large t, the series in n converges extremely rapidly—the logarithmic decrement of damping, for example, increases with increasing term number of the series as n^2, and if we assume that the influence of the inertial terms may be neglected when the sum decreases to $e^{-\pi}$ of its value at $t = 0$, then the following estimate may be given for the time of establishment of the flow (of course, for not too large h):

$$\widetilde{T} = h^2/\pi\nu.$$

The corresponding graph is given in Fig. 1, the time of establishment being expressed in hours.

The most interesting results are contained, however, in the first part of formula (5), denoted below by w_1. This sum is most conveniently analyzed as $h \to \infty$, when (5) passes into the formula for a deep homogeneous sea. In this case, instead of (5) we obtain:

$$w_1 = \frac{1-i}{2\rho\nu}\sum_{k=-\infty}^{\infty} \frac{c_k}{a_2} \exp\{-[i(a_2 z - \sigma_k t) + a_2 z]\}. \tag{6}$$

As can be seen, the steady part of the current can be represented as a superposition of peculiar velocity waves propagating from the sea surface and decaying with depth. The degree of attenuation of each harmonic depends, in particular, on its frequency σ_k. (R. V. Ozmidov [1] drew, in this connection, the conclusion that the attenuation increases as the frequency of the harmonic increases. From what follows, however, it is evident that this conclusion is not exact.)

For the sea surface ($z = 0$), formula (6) gives

$$w_{1k} = \frac{C_k}{2a_2\rho\nu}\left[\sin(\sigma t + 45^\circ) + i\sin(\sigma t - 45^\circ)\right], \tag{7}$$

whence it follows that the direction of rotation of the current vector at the surface coincides with the direction of rotation of the wind vector. But the sum over k contains both positive and negative indices. In the first case, the rotation of the wind and of the current occurs counterclockwise and the angle between them is $+45^\circ$ (the current is deflected to the right of the wind). Conversely, for $k < 0$ the rotation occurs to the left and the angle between the vectors is -45°, if $a > |\sigma_{-k}|$ (in this case the current vector is turned to the left of the wind).

Let us now form the ratios of the moduli of the current velocity at the surface V and of the friction depth D for the k-th harmonic and for the case of a stationary current (Ekman):

$$\frac{V_{1k}}{V_0}\,\frac{D_{1k}}{D}\left(\frac{a}{|a \pm \sigma_k|}\right)^{1/2},$$

where the upper sign under the radical corresponds to $k > 0$, the lower to $k < 0$. As can be seen, when the wind vector rotates counterclockwise, $V_{1k} < V_0$ and $D_{1k} < D$; in the opposite case, $V_{1k} > V_0$ and $D_{1k} > D$.

Of special interest is the case $a = \sigma_{-k}$, in which there arises the extremely curious phenomenon of hydrodynamic resonance in the field of the Coriolis force (akin to that discovered by V. V. Shuleikin [2] in summer-monsoon currents). Setting in formula (4), after substituting into it the expansion for $\tau_1(t)$ and integrating, $a = \sigma_{-k}$, and only then summing the series over n, we obtain for this harmonic:

$$w_{1,-k} = \frac{1}{\rho\nu}\left(\frac{h}{3} - z + \frac{z^2}{2h}\right)^{1/2} C_k e^{i\sigma_{-k}t}.$$

In this case the current at all horizons z is directed along the line of action of the wind, but changes sign at the depth $z \approx 0.42h$, and the Coriolis parameter evidently does not influence the current. However, the "resonant frequency" of the wind depends on latitude, namely:

$$\sigma_{-k} = 2\omega_z \sin\varphi,$$

where σ_{-k} is the frequency of rotation (to the right) of the wind vector; ω_z is the projection of the angular velocity of the Earth' s rotation on the normal to the Earth' s surface, and φ is the latitude of the place. From this formula it follows, in particular, that at a latitude of about 75° resonance occurs when the period of rotation of the wind vector is about 12 hours.

Thus, in the formation of the velocity field of an established wind current, an essential role is played not only by the frequency of variation of the wind speed, but also by the direction of rotation of its vector in time. When the wind vector rotates to the right (in the Northern Hemisphere of the Earth), the phenomenon of hydrodynamic resonance in the field of the Coriolis force may arise.

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Received
13 IX 1960

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[1] R. V. Ozmidov, DAN, **128**, No. 1 (1959). [2] V. V. Shuleikin, Izv. AN SSSR, geophys. ser., No. 6 (1960).

Note: Figure translations are in progress. See original paper for figures.

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