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Abstract

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ON THE THEORY OF DISCRETE SPACE-TIME

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§ 1. We shall arrive at the hypothesis of discrete space-time in the most natural way if we first once again analyze certain fundamentally general features of two modern "nonclassical" theories—quantum mechanics and the theory of relativity. One may say that the constants c and $\hbar$ figuring in these theories appear as a "compensation" for that information which is lost in departing from the classical theory. Namely, c "compensates" the loss of invariance of distances and intervals of time with respect to coordinate transformations; $\hbar$, say, the loss of the possibility of simultaneously measuring the coordinate and momentum of a particle. Such a "compensation" reduces, of course, to the appearance of new effects, essentially nonclassical, which disappear only as $c \to \infty$ and $\hbar \to 0$, respectively. A most important property of the constants c and $\hbar$ is their universality, which makes them "scales of nature": for **all** particles the velocity $\leq c$, and for **all** particles the accuracy of simultaneous measurement of coordinate and momentum is bounded by one and the same $\hbar$.

Let us now suppose that the converse is also true: if a constant is universal, then it must be a "scale of nature" in the spirit of c and $\hbar$ for a quantity of the corresponding dimension, and its appearance is necessarily connected with the loss of some information that existed in the theory without this constant.

Let us examine the "universality" of the constants of field theory. The electric charge, the coupling constant of the strong interaction, and the masses of particles are clearly not universal constants. But Fermi' s constant G for the weak interaction, equal to $1.4 \cdot 10^{-49}$ erg $\cdot$ cm^3, has a universal character, since it is becoming increasingly evident that the intensity of weak processes is the same for all particles (and all particles, except the photon, can participate in weak interactions), if there are no contributions from other interactions. Extracting, as usual, from G the factors $\hbar$ and c, we are in fact dealing with a universal constant of length $l = 7 \cdot 10^{-17}$ cm. The loss of what information is l called upon to "compensate" ? For example, the nonconservation of parity in weak processes. If, however, $l \to 0$, then weak interactions are switched off ($G \to 0$), and parity is again conserved, since in strong and electromagnetic processes this fact has been established experimentally with great accuracy. Thus, l justifies its universality, and we go further, assuming that this constant imposes restrictions on measurements of space–time distances. The only thing that can be assumed here is the impossibility of the existence of intervals smaller than l, i.e., the quantization of 4-space with step l. Thus, the absence of parity conservation in weak interactions is a consequence of the discreteness of space–time. The degree of parity nonconservation in electromagnetic and strong processes is small because their "radii of action" are too large in comparison with l. One may therefore, in a new sense of the word, call these interactions "classical" in relation to the weak ones.

The following obvious requirements are now imposed on the group of motions of 4-space:

1) this group must admit a lesser degree of isotropy of 4-space than the Lorentz group, and therefore, generally speaking, the form $s^2 = x_0^2 - x^2$ ceases to be an invariant of coordinate transformations;
2) the non-invariance of the interval must be the cause of nonconservation of parity;
3) there must necessarily exist a subgroup for which $s^2 =$ invar, so that it would be possible to describe the symmetry of large regions of 4-space.

§ 2. Before studying 4-space all of whose points have integer coordinates, it is useful to recall the analogy that exists between integers and polynomials. Both integers and polynomials can be decomposed into prime factors:

$$a = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_n^{\alpha_n}, \qquad P(t) = (t - a_1)^{\alpha_1} (t - a_2)^{\alpha_2} \dots (t - a_n)^{\alpha_n}.$$

Here p_i are prime numbers, and α_i are the multiplicities of the prime factors. A polynomial can always be expanded into a finite Taylor series in powers of any prime factor $(t - a)$, and the first term of the expansion $P(a)$ is equivalent to $P(t)$ modulo $(t - a)$:

$$P(t) \equiv P(a) \pmod{(t - a)};$$

the second coefficient satisfies the congruence:

$$\frac{P(t) - P(a)}{t - a} \equiv P'(a) \pmod{(t - a)}$$

and so on.

Analogously, a number a can be expanded into a finite series in powers of p (the so-called p-adic series):

$$a = a_0 + a_1 p + a_2 p^2 + \dots + a_N p^N,$$

where

$$a \equiv a_0 \pmod p, \qquad \frac{a - a_0}{p} \equiv a_1 \pmod p$$

and so on.

In other words, any integer a can be regarded as a function of the parameter p, which runs through all prime numbers; the value of this function at the given point p_0 is the residue a modulo p_0. The totality of all integers—the ring C—will then also be a certain "function" of p. But it is known that the residues of all integers modulo a given prime modulus p_0 form the Galois field of order p_0: $GF(p_0)$. Therefore the ring C, considered as a "function" of p, at the point $p = p_0$ has the "value" $GF(p_0)$, or, in "spinor" notation,

$$C = \begin{pmatrix} GF(p_1) \\ GF(p_2) \\ \vdots \\ GF(p_k) \\ \vdots \end{pmatrix} = GF(p); \tag{1}$$

here $p_1 = 2,\ p_2 = 3,\ p_3 = 5$, and so on.

It is clear that the coordinates of 4-space R_4, constructed over the ring C, can likewise be regarded as p-parametrized; then R_4 is the totality of 4-spaces $R_4(p_k)$, analogously to (1). The group of motions (without translations) in the space $R_4(p)$ with fixed p was studied in paper ([1]) and especially fruitfully in paper ([2]). Let us briefly summarize some re-

results (?). For $p = 4n - 1$* the named group consists of transformations that do not change the form $s^2 = x_0^2 - \mathbf{x}^2$ (they themselves form a group!), and of the so-called g-transformations, which change the sign of s^2 to the opposite one: ${s'}^2 = -s^2$. The presence of g-transformations automatically leads to the requirement of the two-component nature of spinor fields, of combined inversion as the only law for the transformation of wave functions under spatial reflections (if time inversion is Wigner' s)**, and even of the $V - A$ variant of four-fermion interaction. The listed symmetry properties of $R_4(p)$, where $p = 4n - 1$, in fact do not depend at all on the particular value of p, since all the fields $GF(p)$ obey the same axioms. Therefore, if in column (1), in place of the rows with $p = 2$ and $p = 4n + 1$, there stood zeros, then we could, roughly speaking, "take out the brackets" these symmetries common to all 4-spaces over the fields $GF(p)$ and regard them as a property of the whole column, i.e. of the present 4-space. This can be achieved by carrying out with the space R_4 a procedure which outwardly resembles renormalization in quantum field theory.

Let a be some integer. If $a \equiv 0 \pmod p$, then this means the divisibility of a by p. In order that a be divisible by 2 and by all p_i having the form $4n + 1$, it is necessary to multiply it by $Z = 2\prod_{i=1}^{\infty} p_i$***. Let $CZ = C'$. Then, obviously, $C \cong C'$, i.e. C' is also a ring. Therefore a 4-space R_4' can be constructed over it. The "quantum" of this space is equal to $l_0 Z = l$, where l_0 is the "quantum" of the space R_4. In order that l be finite, it is necessary to put $l_0 = 0$. We shall regard the space R_4' as "physical." In the "spinor" notation it has the form

$$R_4' = \begin{pmatrix} 0 \\ R_4(3) \\ 0 \\ R_4(7) \\ \vdots \\ R_4(p_i = 4n-1) \\ \vdots \\ \vdots \end{pmatrix}, \tag{2}$$

since the factor Z could not change the value of the "function" R_4 at the points $p = 4n - 1$. In the sense indicated above, the symmetries of the space R_4' are in correspondence with requirements 1)–3).

§ 3. It may be supposed that computations in the present formalism should be carried out according to the following scheme. First it is necessary to obtain the "image" of the result that interests us in the space $R_4(p_0)$, where p_0 is some fixed p of the form $4n-1$, and then, using the fact that, by virtue of the identity of symmetries mentioned above, this "image" is a universal function of p, not depending on its particular value, to average over p in the usual way, i.e. to sum over all $p = 4n - 1$ and divide by the number of these values. If this function does not depend explicitly on p, then immediately after its being obtained in $R_4(p_0)$ it may be regarded as a function in the ordinary discrete space R_4'.

In computations in $R_4(p_0)$ one may at first, in view of the extraordinary similarity of the mathematical apparatus****, transfer there the dynamical principles

* All odd p have the form $4n \pm 1$. In $GF(p)$, for $p = 4n+1$, a field theory cannot be developed because of the imaginary unit contained among the residues. In this case, for example, the meaning of the pseudo-Euclidean interval is lost.

** That is, parity need not be conserved.

*** The fact that Z is equal to infinity should not trouble us, since we are interested in its structure, and that is known. Infinities of this type occur in p-adic expansions of rational numbers.

**** What is meant is the group of motions and the special Fourier analysis developed by E. I. Dolinskii in $R_4(p_0)$.

of the usual theory, for example, diagrammatic techniques. Then for electrodynamics it is natural to postulate Feynman correspondence rules, if one assumes that it is described by a group without g-transformations.

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References

1. H. R. Coish, Phys. Rev., **114**, 383 (1959).

2. I. S. Shapiro, Nucl. Phys. (in press).

Note: Figure translations are in progress. See original paper for figures.