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Fig. 1. Layout of characteristics in an extremal nozzle

Figure 1: Fig. 1. Layout of characteristics in an extremal nozzle

Abstract

Full Text

HYDROMECHANICS

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ON THE BOUNDARY OF THE DOMAIN OF EXISTENCE OF SHOCK-FREE OPTIMAL NOZZLES

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The solution of the variational problem on the best shape of the supersonic part of an axisymmetric jet nozzle is given in papers ([1]-[4]). In constructing the solution, a prescribed bundle of characteristics arising in the flow about a blunt corner A is assumed (Fig. 1). The best contour AB is obtained as the result of solving Goursat' s problem between the characteristic AC and the extremal characteristic CB.

Fig. 1. Layout of characteristics in an extremal nozzle

Let us consider the equations on the extremal CB, given in paper ([4]):

$$m_1 \cos\alpha + 2\pi w \cos(\alpha - \theta) = 0, \tag{1}$$

$$m_2 + 2\pi y \rho w^2 \tan\alpha \sin^2\theta = 0, \tag{2}$$

where α is the angle between the velocity and the characteristic; θ is the angle of inclination of the velocity to the x-axis; w is the velocity; ρ is the density; m_1 and m_2 are constant Lagrange multipliers determined from the results of calculating the bundle of characteristics and from formulas (1) and (2).

From equations (1) and (2) we obtain that on the extremal CB

$$y = -\frac{4\pi m_2}{\rho \sin 2\alpha} \left(\frac{m_1 \sin\alpha \pm \sqrt{4\pi^2 w^2 - m_1^2 \cos^2\alpha}}{m_1^2 - 4\pi^2 w^2} \right)^2, \tag{3}$$

where the plus sign is taken for $\alpha(C) \leqslant \theta(C)$, and the minus sign for $\alpha(C) \geqslant \theta(C)$, owing to the fact that at the point C equation (3) must be satisfied automatically.

As is easy to see, in the case $\alpha(C) > \theta(C)$, along the extremal, as y increases, α and θ decrease; in the case $\alpha(C) < \theta(C)$, along the extremal α increases and θ decreases. At some point of the extremal $\alpha = \theta$. For the further calculation of the extremal it is necessary to change the sign before the radical in formula (3). We note in passing that, as a result of this, the velocity distribution along the extremal characteristic becomes nonmonotone.

It follows from equation (3) that when the point C is moved along the characteristic ACP toward smaller x (in the region $\alpha < \theta$), the derivative in the direction of the extremal $\left.\frac{dy}{d\alpha}\right|_C$ decreases. The value of this derivative passes, at some C_0, through zero and then becomes negative.

Geometrically, this phenomenon is interpreted as a loop at the base of the extremal. Physically it means the impossibility of constructing shock-free solutions of the variational problem for all points of the characteristic ACP (lying to the left of the point C_0) which could serve as initial points for constructing extremals.

If, however, along the extremal, as y increases, α is decreased, then, as calculations show, we arrive at nozzles that do not possess the greatest thrust.

Equating to zero the derivative of the right-hand side of equation (3), using conditions (1) and (2), as well as the known formulas expressing ρ and w in terms of α, we obtain, for a gas flow with constant adiabatic exponent $\varkappa$, the following extremely simple expression relating α and θ on the line AC_0, to the left of which no shock-free solutions of the variational problem exist:

$$\varkappa \sin\theta \sin(\alpha+\theta) - \cos\alpha \sin^2 2\alpha + \sin\theta \sin(\theta-\alpha) - \sin\alpha \sin^2\theta \cos 2\alpha = 0. \quad (4)$$

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Note: Figure translations are in progress. See original paper for figures.