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Abstract

Full Text

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ON THE STABILITY OF RODS AND PLATES UNDER CREEP CONDITIONS

(Presented by Academician Yu. N. Rabotnov on 15 V 1961)

1. Let us consider a hinged rod loaded by a constant longitudinal force T . Let the equation of state have the form ⁽¹⁾

$$\dot{p} = A\sigma^n p^{-\alpha}, \quad p = \varepsilon - \frac{\sigma}{E}. \quad (1)$$

Here σ is the stress, p is the creep strain, and ε is the total strain.

We linearize equation (1) for small deviations of the rod from the rectilinear state:

$$\delta\dot{p} = An\sigma^{n-1}p^{-\alpha}\delta\sigma - A\alpha\sigma^n p^{-\alpha-1}\delta p. \quad (2)$$

Here p, σ refer to the axis of the rod, while $\delta p, \delta\sigma$ denote small increments over the thickness.

Since a rod under creep conditions does not lose stability in the sense of bifurcation, to investigate stability we shall consider the motion of the rod after the action of certain disturbances upon it.

Suppose that at some instant of time a disturbance in the form of a small residual deflection acted on the rectilinear rod. Passing in (2), with the aid of (1), to the variable p and integrating with respect to p , taking into account the initial condition $(\delta p)_{p=p^*} = 0$, where p^* is the creep strain at the instant when the disturbance is applied, we find

$$\delta p = \frac{n}{\sigma} p^{-\alpha} \int_{p^*}^p p^\alpha \delta\sigma dp. \quad (3)$$

Integrating (3) over the thickness of the rod, taking into account the relations

$$\delta\varepsilon = -z(w_{xx} - w_{xx}^0), \quad \int_F z \delta\sigma dF = Tw,$$

we obtain the equation of forced motions

$$EI(w_{xx} - w_{xx}^0) + Tw + \frac{En}{\sigma} T p^{-\alpha} \int_{p^*}^p p_1^\alpha w dp_1 = 0. \quad (4)$$

Putting $w = \tau(p) \sin \frac{\pi x}{l}$, $T_e = \frac{\pi^2 EI}{l^2}$, $\bar{\sigma} = \frac{T}{T_e}$, we shall have the equation for $\tau(p)$

$$(1 - \bar{\sigma})\tau(p) - \tau^0 - \frac{En}{\sigma} \bar{\sigma} p^{-\alpha} \int_{p^*}^p p_1^\alpha \tau(p_1) dp_1 = 0, \quad (5)$$

the solution of which will be

$$\tau = -\frac{\tau^0}{1 - \bar{\sigma}} \left[\left(\frac{p}{p^*} \right)^{-\alpha} e^{k(p-p^*)} + \alpha p^{-\alpha} e^{kp} \int_{p^*}^p p_1^{\alpha-1} e^{-kp_1} dp_1 \right], \quad (6)$$

where

$$k = \frac{\bar{\sigma}}{1 - \bar{\sigma}} \frac{En}{\sigma}.$$

Differentiating (6) twice with respect to time, taking (1) into account and putting $p = p^*$, we find, for the acceleration of the disturbed motion at the initial instant of motion,

$$(\ddot{\tau})_{p=p^*} = A^2 \sigma^{2n} p^{-2\alpha} k \left(k - \frac{2\alpha}{p} \right). \quad (7)$$

It follows from (7) that the character of the disturbed motion depends on the value of p at which the disturbance was applied. If the disturbance acted on the rod at a sufficiently early instant of time, when the creep strain $p < 2\alpha/k$, then the disturbed motion begins with a velocity decreasing in time. For $p > 2\alpha/k$ the disturbed motion proceeds with increasing velocity. We shall call critical the state of a straight rod characterized by the fact that, if a disturbance acted on the rod at that instant, the disturbed motion would occur with nondecreasing velocity, whereas a disturbed motion beginning at any preceding instant of time would occur with decreasing velocity. Thus, for the critical creep strain we obtain

$$p_k = 2\alpha \frac{1 - \bar{\sigma}}{\bar{\sigma}} \frac{\sigma}{En}. \quad (8)$$

The proposed formulation differs from the formulation in ^(1,2) in the character of the forced motions considered and in the formulation of the stability criterion. The value p_k according to formula (8) is twice as large as the value of p_k according to ⁽¹⁾.

The meaning of the proposed stability criterion ($\ddot{\tau} = 0$) becomes clear if one considers the problem of elastic stability from the standpoint of the motion of the rod. For a compressed elastic rod the equation of motion has the form

$$EIw_{xxxx} + Tw_{xx} + \rho F\ddot{w} = 0.$$

Assuming, in the case of hinged supports,

$$w = \tau(t) \sin \frac{\pi x}{l},$$

we obtain

$$\left(1 - \frac{T}{T_e}\right) \tau + \frac{\rho Fl^2}{T_e \pi^2} \ddot{\tau} = 0.$$

Thus, for $T < T_e$ the forced motions are realized with $\ddot{\tau}/\tau < 0$, and for $T > T_e$, $\ddot{\tau}/\tau > 0$.

Obviously, the proposed stability criterion under creep conditions is, in a certain sense, an analogue of the dynamic criterion of elastic stability.

The value of the critical creep strain can also be obtained without seeking the actual expression for the deflection of the disturbed motion (6), i.e., without integrating equation (5). To determine p_k it is sufficient to adjoin to equation (5) and to its two derivatives with respect to p , at $p = p^*$, the condition $(\ddot{w})_{p=p^*} = 0$.

In the more general case, when the equation of state has the form

$$\Phi(\sigma, \rho, \dot{\rho}) = 0,$$

the critical deformation for a longitudinally compressed rod, independently of the support conditions, is determined by the equation

$$\frac{ET}{T - T} = \frac{\dot{\lambda}\nu}{\lambda^2} - \frac{\mu}{\lambda} - \frac{\dot{\nu}}{\lambda},$$

where $\lambda = \Phi_\sigma$, $\mu = \Phi_p$, $\nu = \dot{\Phi}_p$, T is the critical load for elastic loss of stability under the specified support conditions.

2. The formulation of stability problems under creep conditions can be generalized to problems of the stability of plates. Let the equation of state under creep have the form

$$\dot{p}_i = g(p_i, \sigma_i) \sigma_i, \quad (9)$$

and let, between the components of the tensor of creep strain rates \dot{p}_{ij} and the components of the stress deviator σ_{ij}^* , the relations of the flow theory hold:

$$\dot{p}_{ij} = \frac{3}{2} g(p_i, \sigma_i) \sigma_{ij}^*,$$

where

$$p_{ij} = \varepsilon_{ij} - \frac{1}{2G} \sigma_{ij}^*, \quad \sigma_i^2 = \frac{3}{2} \sigma_{ij}^* \sigma_{ij}^*, \quad \dot{p}_i^2 = \frac{2}{3} \dot{p}_{ij} \dot{p}_{ij}.$$

Consider a plate loaded by stresses uniformly distributed through its thickness and not varying with time. Let a disturbance in the form of an initial deflection w^0 have acted on the plate at the instant of time t^* , at which $p_i = p_i^*$. The equation of the disturbed motions is written in the form

$$\begin{aligned} & -e^{-\frac{E}{\sigma_i}(p_i - p_i^*)} \Delta \Delta (w^* - w^0) + \frac{9\sigma_i}{4Eh^2} Pw + \\ & + e^{-\frac{E}{\sigma_i} p_i} \int_{p_i^*}^{p_i} e^{\frac{E}{\sigma_i} p_{i1}} \left[-\frac{\partial}{\partial p_{i1}} \Delta \Delta w - E \frac{\partial g}{\partial p_{i1}} \int_{p_i^*}^{p_{i1}} \frac{1}{g} \left(\frac{1}{g} \frac{\partial g}{\partial \sigma_i} + \frac{1}{\sigma_i} \right) R(w, w^0, p_{i2}) dp_{i2} \right. \\ & \left. - \frac{E}{g} \frac{\partial g}{\partial \sigma_i} R(w, w^0, p_{i1}) \right] dp_{i1} = 0. \end{aligned} \quad (10)$$

Here

$$\begin{aligned} R(w, w^0, p_i) &= e^{-\frac{E}{\sigma_i}(p_i - p_i^*)} \Delta \Delta (w^* - w^0) - \frac{3}{4} e^{-\frac{E}{\sigma_i}(p_i - p_i^*)} PP(w^* - w^0) - \\ & - \frac{9\sigma_i}{4Eh^2} Pw + e^{-\frac{E}{\sigma_i} p_i} \int_{p_i^*}^{p_i} e^{\frac{E}{\sigma_i} p_{i1}} \left[\frac{\partial}{\partial p_{i1}} \Delta \Delta w - \frac{3}{4} \frac{\partial}{\partial p_{i1}} PPw \right] dp_{i1}, \\ Pw &= \alpha_{11} w_{xx} + 2\alpha_{12} w_{xy} + \alpha_{22} w_{yy}, \quad \alpha_{ij} = \frac{\sigma_{ij}}{\sigma_i}, \quad w^* = (w)_{p_i = p_i^*}. \end{aligned}$$

In the case when the variables separate, the expression for the deflections may be sought in the form

$$w(p_i, x, y) = \tau(p_i) w_1(x, y),$$

where w_1 is an eigenfunction of the equation

$$\Delta\Delta w_1 - \frac{9\sigma_i}{4Eh^2}Pw_1 = 0$$

under the specified boundary conditions.

Putting, in equation (10) and in its two derivatives with respect to p , $p_i = p_i^*$, we obtain three equations for τ^0 , τ^* , $(d\tau/dp_i)^*$, $(d^2\tau/dp_i^2)^*$. Adjoin-

Combining with these equations the condition for the stability boundary, which, taking (9) into account, is written in the form

$$\left(\frac{d^2\tau}{dp_i^2}\right)^* + \frac{1}{g} \frac{\partial g}{\partial p_i} \left(\frac{d\tau}{dp_i}\right)^* = 0,$$

we obtain a system of homogeneous equations; setting its determinant equal to zero gives the equation of the stability boundary.

In particular, for $g = A\sigma_i^{n-1}p^{-a}$, for a square simply supported plate uniformly compressed in two directions, we obtain

$$\frac{\sigma_i}{\sigma_{i\exists}} = 1 - \frac{1}{4} \frac{(1+3n)^2}{1+3n^2 + \frac{a\sigma_i}{Ep_i\alpha}(1+6n)}, \quad \sigma_{i\exists} = \frac{8Eh^2\pi^2}{9l^2}.$$

In the case where the variables do not separate, the solution in a neighborhood of the beginning of motion may be sought in the form

$$w(p_i, x, y) = \tau^*w_1(x, y) + (p_i - p_i^*)w_2(x, y) + 1/2(p_i - p_i^*)^2w_3(x, y).$$

In this case, for different points of the plate, generally speaking, their own stability boundary will be obtained. As the stability boundary for the plate as a whole, one may choose the lower envelope of the stability boundaries of all its points.

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