MOTION OF GLACIERS

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Abstract

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GEOPHYSICS

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The motion of glaciers, which distinguishes them from all other accumulations of ice, has been studied since the eighteenth century. At present, the kinematics of the motion and the physical nature of the processes of ice deformation underlying it have been clarified in general terms ([1,2]). However, there is still no satisfactory dynamic theory of motion that would make it possible to obtain analytically a sufficiently accurate characterization of the motion from the physicomechanical properties of ice and the forces acting on it.

With respect to its rheological properties, ice was for a long time considered a viscous body (a Newtonian fluid with viscosity coefficient $\eta \simeq 10^{14}$ poise ([3])). Then an attempt was made to solve the problem of glacier motion by treating ice as a plastic body (a Saint-Venant body with a limiting shear stress of 1 kg cm^{-2} ([4,5])), until, finally, a power-law relation was established between stress and the rate of deformation of ice ([6]). This relation may be represented in the form

$$\dot{\gamma}_i = -\frac{J^2}{\theta - 1}\left(\frac{\tau_i}{k}\right)^n, \tag{1}$$

where τ_i is the intensity of shear stresses; γ_i is the intensity of deformation; θ is the temperature of the ice, and the parameters n and k, judging from the available data of numerous laboratory experiments and measurements of the deformation rate of tunnels and boreholes in glaciers, have on average the values: $n = 3$, $k = 3.1$ kg cm^{-2} yr$^{1/3}$ deg$^{-1/3}$.

Because of the complexity of the problem, all dynamic theories have to introduce a number of simplifying initial conditions. First of all, only steady motion is considered, and accelerations and inertial forces are generally neglected, which is quite permissible at the negligible velocities of motion observed in glaciers. Further, already with less justification, one also has to neglect the elasticity, compressibility, and anisotropy of ice. This makes it possible to assume parallelism of the directions of the principal stresses to the axes of deformation and proportionality of the components of the stress deviator to the components of the rate of deformation.

However, even these simplifications do not make it possible either to determine the complex stress state of ice in a glacier or to integrate the system of differential equations of motion, which in general form take account of the complex stress state. Therefore the solutions ([7,5,8]) are given (or, in any case, are valid) only for laminar flow (pure shear), since they do not take account of normal stresses and the deformations caused by them.* Meanwhile, observations show that this latter type of deforma–

* The only exception is a rigorous solution for the distribution of velocity in the vertical profile of a glacier in the presence, along with shear parallel to the bed, also of uniaxial longitudinal deformation ([9]). But this solution assumes that the velocity at the surface is known, that the gradient of its variation in the direction of motion is known and constant, and that the longitudinal deformation is plane, which greatly limits the possibility of applying this solution.

plays an important, and often the principal, role in the motion of glaciers, and its omission leads to the fact that the existing dynamic theories of motion in most cases give quite unsatisfactory results.

Therefore, instead of discarding normal stresses and strains, the author, as a first approximation, proposed a solution of the problem of glacier motion based on the assumption of independence of normal and shear stresses. This assumption makes it possible to consider separately the stress-strain states of extension–compression $[\sigma_1, \sigma_2, \sigma_3]$, $[\varepsilon_1, \varepsilon_2, \varepsilon_3]$ and of pure shear $[\pm\tau, 0, \mp\tau]$, $[\pm{}^1/_2\gamma, 0, \mp{}^1/_2\gamma]$, whose axes are parallel and perpendicular to the glacier bed.

Let us take a right rectangular coordinate system with its origin at the head of the glacier (or on the surface at the center of the glacier shield), with the horizontal axis x directed along the line of motion and the z axis directed downward. Introduce the notation: z_n and z_l are the ordinates of the surface and the bed of the glacier; $Z = z_l - z_n$ is the glacier thickness; $\zeta = z - z_n$ is the depth below the glacier surface; α and β are the angles of inclination of the surface and the bed of the glacier to the x axis; ρ is the density of ice; g is the acceleration due to gravity; u, v, w are the components of the velocity of motion of a point in the directions of the coordinate axes.

We shall first confine ourselves to the plane problem ($\sigma_2 = \sigma_y = {}^1/_2[\sigma_1 + \sigma_3]$, $\varepsilon_2 = \dot{\varepsilon}_y = 0, v = 0$). The component p_x of the force of an elementary column of ice of dimensions $dx\,dy\,Z$ in the x direction is equal to

$$p_x = \rho g\, dx\, dy\, Z \cos^2\beta \left(\operatorname{tg}\alpha + Z\cos^2\beta\, \operatorname{tg}\beta\, \frac{\partial \operatorname{tg}\beta}{\partial x}\right). \qquad (2)$$

The horizontal component $\tau_{zx}(z_l)$, parallel to the bed, of the shear stress at the bed, equal to the stress of the force of external friction, is expressed as:

$$\tau_{zx}(z_l) = \rho g\, Z \cos^2\beta\, f, \qquad (3)$$

where f is the coefficient of friction of the glacier on its bed. In a vertical section of the glacier the shear stress varies proportionally to the depth below the surface ζ. Since the head of the glacier is fixed, the horizontal component

$\bar{\sigma}'_x = {}^1\!/_2(\sigma_x - \sigma_z)$ of the mean deviator of the normal stress along the x axis in the given transverse section of the glacier x is equal to one half of the excess of the force of the underlying part of the glacier over the force of external friction, referred to a unit area of the transverse section:

$$\bar{\sigma}'_x(x) = \frac{1}{Z}\int_x^l \rho g\, Z\cos^2\beta \left(\operatorname{tg}\alpha + Z\cos^2\beta\,\operatorname{tg}\beta\,\frac{\partial\operatorname{tg}\beta}{\partial x} - f\right)dx, \qquad (4)$$

where l is the abscissa of the end of the glacier and Z, α, β, and f are functions of x.

If differences in the temperature of the ice are neglected, then from (1), (3), and (4) we obtain the following values of the strain rates:

$$\dot{\gamma}_{zx}(z) = -\frac{2}{\theta - 1}\left(\frac{\rho g}{k}\zeta\cos^2\beta\, f\right)^n, \qquad (5)$$

$$\dot{\varepsilon}_x(x) = -\dot{\varepsilon}_z(x) =$$

$$= -\frac{1}{\theta - 1}\left[\frac{1}{2kZ}\int_x^l \rho g\, Z\cos^2\beta \left(\operatorname{tg}\alpha + Z\cos^2\beta\,\operatorname{tg}\beta\,\frac{\partial\operatorname{tg}\beta}{\partial x} - f\right)dx\right]^n, \qquad (6)$$

where $\bar{\theta}$ is the mean temperature of the glacier.

The horizontal component $\bar{u}(x)$ of the mean velocity of motion of the given vertical section of the glacier x is equal to the sum of the horizontal components-

of the absolute velocities $\bar{\dot{\varepsilon}}_x dx$ of longitudinal extensions and compressions experienced by the ice over the entire distance from the beginning of the glacier to the point x:

$$\bar{u}(x) = \int_0^x \bar{\dot{\varepsilon}}_x(x)\, dx. \qquad (7)$$

The deviations of the velocities of motion at different depths from the mean velocity are due to differences in the velocities of laminar flow $u_{\text{lam}}(z)$, which is a component of the complex motion of the glacier:

$$u_{\text{lam}}(z) = \int_z^{z_1} \dot{\gamma}_{zx}(z)\, dz = -\frac{1}{n+1}\frac{2}{\bar{\theta} - 1}\left(\frac{\rho g}{k}\cos^2\beta f\right)^n \left(Z^{n+1} - \zeta^{n+1}\right). \qquad (8)$$

The mean velocity $\bar{u}_{\text{lam}}$ of the laminar flow is equal to:

$$\bar{u}_{\text{lam}} = \frac{1}{Z}\int_{z_{\text{p}}}^{z_1} u_{\text{lam}}(z)\,dz = -\frac{1}{n+2}\frac{2}{\bar{\theta}-1}\left(\frac{\rho g}{k}\cos^2\beta f\right)^n Z^{n+1}. \tag{9}$$

The velocity $u_{\text{gl}}(x)$ of glacier sliding along the bed, which earlier theories groundlessly set equal to zero, or left unaccounted for, is determined as the difference between the mean velocity of motion of the given cross section of the glacier and its laminar component:

$$u_{\text{gl}}(x) = \bar{u}(x) - \bar{u}_{\text{lam}}(x) = \bar{u}(x) + \frac{1}{n+2}\frac{2}{\bar{\theta}-1}\left(\frac{\rho g}{k}\cos^2\beta f\right)^n Z^{n+1}. \tag{10}$$

In many cases the sliding velocity along the bed reaches 50–100% of the total velocity of glacier motion. From (8) and (10), the horizontal component of the entire velocity of glacier motion at depth ζ below the surface is equal to

$$u(x,z) = \bar{u}(x) - \frac{1}{n+1}\frac{2}{\bar{\theta}-1}\left(\frac{\rho g}{k}\cos^2\beta f\right)^n \left(\frac{1}{n+2}Z^{n+1} - \zeta^{n+1}\right). \tag{11}$$

The horizontal and vertical strain rates of the ice at any depth below the surface are determined from (11) by differentiating the horizontal velocity of motion with respect to x:

$$\dot{\varepsilon}_x(x,z) = -\dot{\varepsilon}_z(x,z) = \bar{\dot{\varepsilon}}_x + \frac{2}{\bar{\theta}-1}\left(\frac{\rho g}{k}\cos^2\beta f\right)^n \left[\left(\frac{1}{n+2}Z^n - \zeta^n\right)\right.$$
$$\left.\times\left(\operatorname{tg}\alpha - \operatorname{tg}\beta\right) - \frac{n}{n+1}\left(\frac{1}{n+2}Z^{n+1} - \zeta^{n+1}\right)\left(\frac{\partial f/\partial x}{f} - 2\cos^2\beta\,\operatorname{tg}\beta\,\frac{\partial\,\operatorname{tg}\beta}{\partial x}\right)\right]. \tag{12}$$

The vertical component $w(x,z)$ of the velocity of glacier motion at the point (x,z) is the algebraic sum of the vertical velocity of displacement during motion parallel to the bed, due to the inclination of the latter, and the integral of the rate of vertical deformation of the ice from the given point to the bed *:

$$w(x,z) = u(x,z)\operatorname{tg}\beta - \int_z^{z_1}\dot{\varepsilon}_z(x,z)\,dz = u(x,z)\operatorname{tg}\beta + \bar{\dot{\varepsilon}}_x(Z-\zeta) -$$
$$-\frac{2}{\bar{\theta}-1}\left(\frac{\rho g}{k}\cos^2\beta f\right)^n\left\{\left[\frac{1}{(n+1)(n+2)}Z^{n+1} + \zeta\left(\frac{1}{n+2}Z^n - \frac{1}{n+1}\zeta^n\right)\right]\right.$$
$$\left.\times\left(\operatorname{tg}\alpha - \operatorname{tg}\beta\right) - \frac{n}{(n+1)(n+2)}\zeta\left(Z^{n+1} - \zeta^{n+1}\right)\left(\frac{\partial f/\partial x}{f} - 2\cos^2\beta\,\operatorname{tg}\beta\,\frac{\partial\,\operatorname{tg}\beta}{\partial x}\right)\right\}. \tag{13}$$

* The actual velocity of vertical motion differs from that described by equation (13) because of compaction of the ice with depth. For a method of taking this factor into account, see ([10]).

By differentiating (6) we obtain the equation

$$g\cos^2\beta\left(\operatorname{tg}\alpha + Z\cos^2\beta\operatorname{tg}\beta\frac{\partial\operatorname{tg}\beta}{dx} - f\right) = -(\bar{\theta}-1)^{1/n}k\dot{\bar{\varepsilon}}^{1/n}\left(\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{Z} - \frac{1}{n}\frac{\partial\dot{\bar{\varepsilon}}_x/\partial x}{\dot{\bar{\varepsilon}}_x}\right), \tag{14}$$

which relates the physico-mechanical properties of ice, the stress due to gravity, the thickness, shape, deformation rate, and coefficient of friction of a glacier against its bed in a given vertical section. If the glacier is in a stationary state, then the equality holds

$$\rho Z\bar{u} = \int_0^x a(x)\,dx, \tag{15}$$

where a is the balance of mass on the upper and lower boundaries of the elementary ice column. From (14) and (15) we obtain the differential equation of a stationary glacier:

$$\rho g\cos^2\beta\left(\operatorname{tg}\alpha + Z\cos^2\beta\operatorname{tg}\beta\frac{\partial\operatorname{tg}\beta}{dx} - f\right) = -\frac{1}{n}(\bar{\theta}-1)^{1/n}k\frac{1}{Z^{1/n}}\left[\frac{a}{\rho}+\bar{u}(\operatorname{tg}\alpha-\operatorname{tg}\beta)\right]^{1/n}\times$$

$$\times\left[\frac{\frac{e}{\rho}\operatorname{tg}\alpha - \bar{u}\left(\frac{\partial\operatorname{tg}\alpha}{dx}-\frac{\partial\operatorname{tg}\beta}{dx}\right)}{\frac{a}{\rho}+\bar{u}(\operatorname{tg}\alpha-\operatorname{tg}\beta)} + (n-2)\frac{\operatorname{tg}\alpha-\operatorname{tg}\beta}{Z}\right], \tag{16}$$

where $e = -\partial a/\partial z$ is the "energy of glaciation" ([11]).

In the nonstationary state, the rate of change of the thickness and height of the glacier surface, according to (15), is

$$\frac{dZ}{dt} = -\frac{dz_n}{dt} = \frac{a}{\rho} + \bar{u}(\operatorname{tg}\alpha-\operatorname{tg}\beta) - \dot{\bar{\varepsilon}}_x Z, \tag{17}$$

and the rate of change of the angle of slope of the surface is

$$\frac{d\operatorname{tg}\alpha}{dt} = \frac{e}{\rho}\operatorname{tg}\alpha + Z\frac{\partial\dot{\bar{\varepsilon}}_x}{dx} - 2\dot{\bar{\varepsilon}}_x(\operatorname{tg}\alpha-\operatorname{tg}\beta) - \bar{u}\left(\frac{\partial\operatorname{tg}\alpha}{dx}-\frac{\partial\operatorname{tg}\beta}{dx}\right). \tag{18}$$

The relations presented make it possible to analyze the interrelations between dynamic characteristics, morphology, climatic conditions, and the intensity of geological activity of glaciers, and also to determine the direction and rate of their evolution.

Under certain additional assumptions an approximate solution of the spatial problem, which is of considerably greater complexity, is also given.

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Note: Figure translations are in progress. See original paper for figures.