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L. I. RUDAKOV and R. Z. SAGDEEV

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Abstract

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ON THE INSTABILITY OF AN INHOMOGENEOUS RAREFIED PLASMA IN A STRONG MAGNETIC FIELD

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Experimental investigations of ohmic heating of a plasma by a current directed along a strong magnetic field ([1,2]) show that instabilities develop in the plasma which cannot be explained within the framework of ideal magnetohydrodynamics. In works ([2,3]) an interpretation of these phenomena is proposed on the basis of kinetic theory as the excitation, by electrons carrying the current along the magnetic field, of the so-called "ion-sound" oscillations. The occurrence of such an instability, however, is possible only in a strongly nonisothermal plasma, in which the electrons are considerably hotter than the ions,

$$T_e \gg T_i$$

(T is the temperature).

In ([4]) a mechanism of instability is proposed within the framework of hydrodynamics, but with account taken of the finiteness of the conductivity. Under conditions when the mean free path of the electrons is comparable with the characteristic dimensions of the apparatus, the role of this instability decreases.

Below we consider a mechanism of instability whose occurrence is not connected with the condition of nonisothermality of the plasma or with the obligatory presence of a longitudinal electric current.

Let us adopt the following assumptions: 1) the plasma pressure is small in comparison with the magnetic pressure, $p \ll H^2/8\pi$; 2) the instability develops in a time shorter than the collision time; 3) the frequency of the oscillations being excited is much smaller than the ion cyclotron frequency, $\omega \ll eH/Mc$, and the wavelengths of the perturbations λ are much larger than the ion Larmor radius, $\lambda \gg r_H$; 4) $H^2/8\pi \ll nMc^2$.

Let the magnetic field **H** be everywhere directed along the z-axis, and let the quantities characterizing the stationary state of the plasma vary in the x direction. We shall consider small perturbations of the stationary state having the form

$$A(x) \exp i(k_z z + k_y y - \omega t).$$

Then the corrections to the electron and ion distribution functions, found from the solution of the linearized kinetic equation (for electrons and ions, respectively), will have the form ([5])

$$f_\alpha = -i \left(\frac{e_\alpha}{m_\alpha} E_z \frac{\partial f_{0\alpha}}{\partial v_z} + c \frac{E_y}{H_0} \frac{\partial f_{0\alpha}}{\partial x} \right) \frac{1}{\omega - k_z v_z}; \tag{1}$$

$\alpha = i, e$ (electrons, ions); f_0 is the unperturbed distribution. Here condition 3) has been essentially used; when it is satisfied, the motion of particles across the lines of force of the magnetic field is a drift motion:

$$\mathbf{V}_\perp = c[\mathbf{E} \times \mathbf{H}]/H^2.$$

Since frequencies smaller than the ion Larmor frequency are considered and, consequently, certainly smaller than the ion Langmuir frequency $(4\pi n e^2/M)^{1/2}$, the plasma may be regarded as quasineutral. Then, in the equation relating the time variation of the space-charge density ρ to the current density $\mathbf{j}$: $\partial\rho/\partial t + \operatorname{div} \mathbf{j} = 0$, the first term may be omitted:

$$\operatorname{div} \mathbf{j} = 0. \tag{2}$$

The perturbation of the current density $\mathbf{j}$ is expressed as follows: the longitudinal component, by definition, has the form $j_z = e_a \int v f_a \, dv$; the transverse component, in the case of a strong magnetic field and low frequencies (condition 3), can be expressed directly in terms of the electric field by means of the so-called "static" dielectric permittivity $\varepsilon_\perp = 4\pi n M c^2/H^2$.

$$\mathbf{j}_\perp = \frac{\varepsilon_\perp}{4\pi} \frac{\partial \mathbf{E}_\perp}{\partial t} = -\frac{i\omega}{4\pi} \varepsilon_\perp \mathbf{E}_\perp. \tag{3}$$

Finally, in the problem of interest to us one more simplification may be introduced: the electric field may be considered potential, $\mathbf{E} = -\vec{\nabla}\varphi$. This is valid when the inequality $\mathbf{E} \gg \frac{1}{c}\omega \mathbf{A}$ is satisfied, or (expressing $\mathbf{A}$ in terms of the current density $\mathbf{j}$ from the equation $\Delta \mathbf{A} = \frac{4\pi}{c}\mathbf{j}$)

$$\mathbf{E} \gg \frac{4\pi}{c^2} \frac{\mathbf{j}}{k^2}. \tag{4}$$

For the transverse components, using relation (3), inequality (4) can be reduced to the following:

$$\frac{\omega^2}{k^2} \ll \frac{H^2}{4\pi n M}.$$

This means that we restrict ourselves to perturbations propagating with phase velocities much smaller than the Alfvén velocity. The longitudinal component of (4), as an estimate shows, whose derivation we do not give (it is more cumbersome), reduces to the condition

$$\frac{\omega^2}{k_z^2} \ll \frac{H^2}{4\pi n M}.$$

Now, using equations (1)–(3) and the condition that the electric field is potential, after simple calculations we obtain the equation for the perturbation of the scalar potential φ

$$\frac{d^2\varphi}{dx^2} - F(\omega, k, x)\varphi = 0, \tag{5}$$

where

$$F(\omega, k, x) = \frac{\omega_H^2}{\omega} k_z^2 \int \frac{v_z\, dv_z}{\omega - k_{zv} z} \left\{ \left[\frac{M}{m} \frac{\partial f_{oe}}{\partial v_z} + \frac{\partial f_{oi}}{\partial v_z} \right] + \frac{k_y}{k_z} \frac{1}{\omega_H} \frac{\partial}{\partial x} \left(f_{oi} - f_{oe} \right) \right\}$$

$$\left(\omega_H = \frac{eH}{Mc} \right).$$

(When taking the integrals in this expression, the pole of the integrand must be bypassed from below.)

In a homogeneous plasma, (5) gives the well-known dispersion equation for "ion-sound" oscillations

$$\int \frac{v_z\, dv_z}{\omega - k_{zv} z} \left[\frac{M}{m} \frac{\partial f_{oe}}{\partial v_z} + \frac{\partial f_{oi}}{\partial v_z} \right] = 0.$$

In an inhomogeneous plasma, however, one must seek solutions of equation (5) for φ that decrease in both directions (as $x \to \pm\infty$). Together with this requirement, equation (5) determines the eigenvalues ω.

Solutions of this type are local solutions near the point x, where

$$F(\omega, k, x) = 0. \tag{6}$$

In the neighborhood of such x, equation (5) generally takes the form of an Airy equation with a complex argument (since ω is a complex quantity). Such an equation has solutions that decay on both sides of the point where $F(\omega,k,x)=0$. (This point is analogous to a "turning point" in the Schrödinger equation for one-dimensional motion.)

In equation (6), which plays the role of a "dispersion" equation also in an inhomogeneous plasma, the term containing $\frac{\partial}{\partial x}(f_{0i}-f_{0e})$ can become substantial even for weak inhomogeneity if $k_y \gg k_z$.

Application of the conclusions presented above to the case where the velocity distribution of the particles is Maxwellian,

$$f_{0\alpha}=\frac{1}{\sqrt{2\pi T/m_\alpha}}\,e^{-m_\alpha v_z^2/2T}\qquad (T_i=T_e=T)$$

(n and T are functions of the coordinate x), leads to the following results:

1. If $n(x)=\text{const}$, $T(x)\neq\text{const}$, then for frequencies in the interval

$$\sqrt{\frac{T}{M}}\ll\frac{\omega}{k}\ll\sqrt{\frac{T}{m}}$$

and under the condition

$$\frac{k_y^2}{k_z^2}\left(\frac{d\ln T}{dx}\right)^2 r_H^2\gg 1,$$

equation (4) takes the form

$$\omega^3+\frac{k_y}{k_z}\frac{d\ln T}{dx}\,k_z^3\,\frac{2T^2}{M^2\omega_H}=0. \tag{7}$$

Here there is always a root

$$\omega=\frac{1}{\sqrt{3}}(1+2i)\left(\frac{k_y}{k_z}\frac{d\ln T}{dx}\,k_z^3\,\frac{2T^2}{M^2\omega_H}\right)^{1/3},$$

which gives instability. Let us note that the restriction to the frequency interval

$$\sqrt{\frac{T}{M}}\ll\frac{\omega}{k}\ll\sqrt{\frac{T}{m}},$$

as is known from the theory of ion oscillations of a homogeneous plasma, is equivalent to the hydrodynamic approximation, in which the adiabatic exponent γ for the electron gas is equal to 1. It can be shown that the results of this item, also for an inhomogeneous plasma, are contained in the hydrodynamic approach. Accordingly, equation (7) does not exhaust all the roots of the dispersion equation (6).

2. If $T(x) = \text{const}$, $n(x) \neq \text{const}$, an investigation of equation (6) shows that the complex roots present correspond to damping of the oscillations (the plasma is stable).

The instability at variable temperature can be interpreted in the following intuitive way. In a homogeneous plasma, an "ion electrostatic" wave is an oscillation (sound) propagating along the lines of force of the magnetic field $\mathbf{H}$. In an inhomogeneous plasma ($T(x) \neq \text{const}$), in an "oblique" wave (k_z, $k_y \neq 0$) the transverse motion with velocity $c[\mathbf{E} \times \mathbf{H}]/H^2$ leads to heat transport. One can choose the direction of propagation of the wave, the sign of k_y/k_z, in such a way that a continuous influx of heat from the region with the higher unperturbed temperature enters the phase of "compression" of the plasma in the wave, where the temperature increases. This will be the cause leading to the growth of the oscillations.

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CITED LITERATURE

1. R. A. Ellis, L. P. Goldberg, I. G. Gorman, Phys. Fluids, **3**, 468 (1960).
2. L. Spitzer, Phys. Fluids, **3**, 659 (1960).
3. I. B. Bernstein, E. A. Frieman et al., Phys. Fluids, **3**, 136 (1960).
4. B. B. Kadomtsev, A. V. Nedospasov, J. Nuclear Energy, Part C, **1**, 230 (1960).
5. L. I. Rudakov, R. Z. Sagdeev, ZhETF, **37**, 1337 (1959).

Note: Figure translations are in progress. See original paper for figures.