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**Abstract**

**Full Text**

## **Reports of the Academy of Sciences of the USSR**

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### **GEOPHYSICS**

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## **APPLICATION OF ELLIPTICALLY POLARIZED RADIO WAVES TO THE STUDY OF CLOUDS AND PRECIPITATION**

*(Presented by Academician E. K. Fedorov, 31 III 1961)*

With the aid of a radar station that transmits and receives, on a single antenna, a linearly polarized wave, it is possible to obtain information on the microstructure of a meteorological object under the assumption that the scattering particles are spheres. It is known, however, that many particles of clouds and precipitation—ice crystals, large drops, hail—do not have a spherical shape. Therefore the information on such particles obtained when linear polarization is used proves to be far from complete.

The effective surface of nonspherical particles is determined not only by their volume and dielectric permittivity, but also by the shape and orientation of the particles relative to the plane of polarization of the radiated field. For linear polarization and an antenna system capable of receiving the depolarization component, these questions were considered in work (2) and partially in (3).

Let us examine the general case of the meteorological application of elliptically polarized radio waves, the transmission and reception of which can be carried out by a single antenna, and show the possibility of obtaining additional information about the scattering object.

In doing so, the following assumptions are introduced:

1. Nonspherical raindrops, hail, and disk-shaped and rod-shaped ice crystals are approximated by ellipsoids of revolution with shape coefficient  $\rho$ , equal to the ratio of the ellipsoid diameter to its axis of revolution.
2. Only dipole scattering is considered.

The admissibility of such a model has recently been confirmed experimentally (1).

I. Let us first consider scattering by a single arbitrarily oriented ellipsoid.

The radar equation for a unit target, with allowance for polarization, may be written in the general form <sup>(4)</sup>

$$P_r = \frac{P_t G^2 \lambda^2}{(4\pi)^3 R^4} \sigma \eta \eta^*, \quad (1)$$

where the polarization characteristic is  $\eta = \overline{(AE_1)^* E_2}$ ;  $A$  is the scattering matrix relating the components of the scattered field to the components of the incident wave;  $E_1$  is the unit column vector of the transmitted wave,  $E_2$  that of the received wave; the asterisk denotes complex conjugation, and the bar transposition.

In the general case of an elliptically polarized wave, the electric-field vector is characterized by the column

$$\mathbf{E} = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \beta_1 e^{i\varphi_1} \\ \beta_2 e^{i\varphi_2} \end{pmatrix},$$

and the vectors  $AE_1$  and  $E_2$  are complex.

The value of the matrix  $A$  can be found from the expression for the field scattered by an electric dipole <sup>(5)</sup>:

$$\mathbf{E} = \frac{1}{c^2 R} [[\ddot{\mathbf{P}}\mathbf{n}]\mathbf{n}], \quad (2)$$

where  $\mathbf{P}$  is the dipole moment,  $\mathbf{n}$  is the scattering direction. With the aid of the transformation matrix  $B$  from the radiation coordinate system  $x, y, z$  to the coordinate system  $\xi, \eta, \zeta$  associated with the ellipse, we find

$$\mathbf{P}_{x,y,z} = \frac{v}{4\pi} B^{-1} G B \mathbf{E}_{x,y,z}, \quad (3)$$

where  $v$  is the volume of an ellipsoid with dielectric permittivity  $\varepsilon$ ;  $G$  is the polarizability tensor with elements  $g(\varepsilon, \rho)$ ,  $g'(\varepsilon, \rho)$ ,  $g''(\varepsilon, \rho)$ .

From (3) and (2) we have  $A = B^{-1} G B$ .

The elements of the matrix  $B$  are direction cosines; therefore, taking into account the relations between them, we obtain

$$A = \begin{pmatrix} a & b \\ b & c \end{pmatrix}, \quad (4)$$

where  $a = \alpha_1^2(g - g') + g'$ ,  $b = \alpha_1 \alpha_2(g - g')$ ,  $c = \alpha_2^2(g - g') + g'$ ;  $\alpha_1$  and  $\alpha_2$  are the cosines of the angles formed by the axis of rotation of the ellipsoid with the axes  $Ox$  and  $Oy$  (the values of these elements are given in <sup>(2)</sup>).

Fig. 1

Figure 1: Fig. 1

**Fig. 1.** Dependence of the echo-signal power for elliptical polarization on the polarization parameter (for rain and  $\delta = 0$ )

It is now not difficult to obtain the magnitude of the echo signal from an ellipsoidal particle under elliptical polarization. Let  $\alpha$  be the angle between the optical axis of the polarizing device, which transforms linear polarization into elliptical polarization, and the electric vector of the linear polarization  $\mathbf{E}$ ;  $\varphi$  is the phase shift introduced by this device. Then the radiated elliptical field can be written as

$$\mathbf{E}_1 = \begin{pmatrix} \cos \alpha \\ \sin \alpha \cdot e^{-i\varphi} \end{pmatrix}. \quad (5)$$

Hence, fixing one of the parameters, for example for the polarization factor in (1), we obtain

$$I = \eta\eta^* = (a \cos^2 \alpha - c \sin^2 \alpha)^2 + b \sin^2 2\alpha. \quad (6)$$

In particular, for  $\alpha = 0$  and  $\pi/2$  the polarization is linear, and for  $\alpha = \pi/4$  it is circular.

II. Further, the magnitude of the echo signal from clouds and precipitation consisting of a multitude of incoherent ellipsoidal scatterers will be

$$P_{r\text{ell}} = c \sum_{v, \alpha_1, \alpha_2} N(v)v^2 N_1(\alpha_1, \alpha_2) I, \quad (7)$$

where  $c$  is a coefficient depending on the parameters of the station and on the range of the meteorological object;  $N(v)$  is the concentration of particles with volume  $v$ ;  $N_1(\alpha_1, \alpha_2)$  is the distribution function of the particles with respect to orientations.

If it is assumed that the expressions under the summation sign are independent, the mean values will refer to the particles making the principal contribution to the magnitude of the echo signal.

Let us consider three particular cases of the behavior of the function  $N_1(\alpha_1, \alpha_2)$ .

1. Vertical orientation, i.e., the axis of rotation of the ellipsoid is vertical. This includes precipitation in the form of rain or hail. Introducing  $\delta$ —the antenna-site angle—from (6) and (7), accurate to a constant, we obtain (see Fig. 1):

$$P_{r\text{ell}} \sim \frac{(g-g')^2}{4} \sin^4 2\alpha \cos^2 \delta + [\cos^2 \delta (g-g') + g']^2 \cos^2 2\alpha. \quad (8)$$

2. Horizontally random orientation, i.e., the axis of rotation is randomly situated in the horizontal plane. In this case, for vertical sounding  $\delta = \pi/2$ , and we have

$$P_{r\text{ell}} \sim \sin^2 2\alpha \left[ \frac{(g-g')^2}{16} - \frac{gg'}{2} \right] + (\cos^4 \alpha + \sin^4 \alpha) \left[ \frac{3}{8}(g-g')^2 + gg' \right]. \quad (9)$$

3. Spatially random orientation, i.e., any position of the axis in space is equally probable. Cases 2 and 3 occur mainly in clouds. With arbitrary orientation, the power of the echo signal does not depend on the direction of radiation (see Fig. 2)

$$P_{r\text{ell}} \sim \sin^2 2\alpha \left[ \frac{(g-g')^2}{30} - \frac{(g-g')g'}{3} - \frac{g'^2}{2} \right] + (\cos^4 \alpha + \sin^4 \alpha) \left[ \frac{(g-g')^2}{5} + \frac{2}{3}(g-g') + g'^2 \right]. \quad (10)$$

**Fig. 2.** Dependence of the ratio of the echo-signal magnitudes for circular and linear polarizations on the shape factor  $\rho$  for vertically oriented particles

From (8), (9), and (10) it is evident that by changing the polarization parameters one can obtain additional information on the shape or orientation of particles and, in a number of cases, since the elements of the polarizability tensor are sensitive to  $\varepsilon$ , also on their aggregate state. This information will prove useful for radar separation of the droplet and crystalline phases in clouds, recognition of hail, monitoring under artificial interventions, and other applications. In addition, changes in the function  $N_1(\alpha_1, \alpha_2, t)$ , for example when artificial targets are introduced into the atmosphere, may be associated with processes of small-scale turbulence.

**Fig. 3.** Example of a recording of the echo signal from precipitation and a “bright band” when the polarization parameter is changed

- III. To realize some of the indicated possibilities, at the Central Aerological Observatory an installation was created at a centimeter-wave vertical-sounding radar station with a polarization device, by means of which emission of any type of polarization is carried out. In addition, an auxiliary antenna received the depolarization component. The ratio of echo signals at different polarizations was read directly from an attenuator placed in the high-frequency path.

We present part of the results of measurements of the quantity  $P_{r\text{ell}}/P_{r\text{lin}}$  at  $\alpha = \pi/4$  and  $\alpha = 0$ , performed in precipitation and in the layer of the “bright band.”

In snowfalls, the values of  $P_{r\text{ell}}/P_{r\text{lin}}$  were, as a rule, within the range from  $-4$  to  $-11$  dB, with the smaller value for dry snowflakes and the larger for water-coated large flakes. The mean value,  $-7$  to  $-8$  dB, corresponds to a model of disk-like shape with  $\rho \geq 10$ .

If the particles were arbitrarily oriented in space, then, according to calculations, the ratio of the depolarization component to the echo signal for circular polarization should have been  $1 : 2$ . Measurements in a number of cases give  $1 : 3$  and  $1 : 4$ . This means that snow crystals have a certain preferred orientation in the vertical plane. In rains, with vertical sounding, the quantities under consideration are significantly smaller— $-16$  to  $-23$  dB, down to the limits of the intrinsic depolarization in the antenna system (7). The shape factor here is close to unity.

In the “bright band,” measurements under different conditions do not yield a stable value of  $P_{r\text{ell}}/P_{r\text{lin}}$ . This ratio may be rather large, approaching the value for wet snow (Fig. 3), and, in some cases, small, differing by only a few decibels from its value for liquid precipitation. Evidently, changes in the shape and orientation of particles during melting substantially complicate the picture corresponding to the usual conception of the “bright band.”

A discussion of the results cannot be carried out within the scope of the present article, but the quantities we have obtained testify to the sensitivity of the polarization method to the shape, orientation, and phase state of the particles. It may be assumed that the use of elliptical polarization will significantly broaden the program of meteorological investigations by means of radar equipment.

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