Mechanics

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Abstract

Full Text

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ON ONE REGULAR PRECESSION OF A RIGID BODY HAVING CAVITIES FILLED WITH FLUID

(Presented by Academician L. I. Sedov, 29 III 1961)

In paper ([1]) one new particular solution was indicated of the Euler–Poisson equations

$$\begin{aligned} A\frac{dp}{dt} - (B-C)qr &= Mg(z_0\gamma' - y_0\gamma''), & \frac{d\gamma}{dt} &= r\gamma' - q\gamma'', \\ B\frac{dq}{dt} - (C-A)pr &= Mg(x_0\gamma'' - z_0\gamma), & \frac{d\gamma'}{dt} &= p\gamma'' - r\gamma, \\ C\frac{dr}{dt} - (A-B)pq &= Mg(y_0\gamma - x_0\gamma'), & \frac{d\gamma''}{dt} &= q\gamma - p\gamma', \end{aligned} \tag{1}$$

which holds under the following conditions imposed on the parameters of the rigid body:

$$C > 2A > 2B, \qquad y_0 = 0, \qquad x_0 \sin\alpha = z_0 \cos\alpha. \tag{2}$$

The indicated solution has the form

$$p = \frac{v}{A}(\cos\rho + \chi\sin\rho\cos\sigma), \quad r = \frac{v}{C}(\sin\rho - \chi\cos\rho\cos\sigma), \quad q = v\varkappa\sin\sigma,$$

$$\gamma = \frac{B(A+C)\cos\rho + 3AC\chi\sin\rho\cos\sigma + 3A(C-B)\cos\rho\sin^2\sigma}{3AC - B(A+C)}, \tag{3}$$

$$\gamma' = \frac{3ABC\,\varkappa\sin\sigma - 3\frac{\chi}{\varkappa}(C-A)\sin\rho\cos\rho\sin\sigma\cos\sigma}{3AC - B(A+C)},$$

$$\gamma'' = \frac{B(A+C)\sin\rho - 3AC\chi\cos\rho\cos\sigma + 3A(A-B)\sin\rho\sin^2\sigma}{3AC - B(A+C)},$$

where σ is a function of time determined from the equation

$$\frac{d\sigma}{dt} = \frac{v}{N}(k + k' \cos\sigma).$$

The various constants entering into (2), (3) have the following values:

$$\cos\alpha = \frac{1}{H}\sqrt{C(A-B)(C-2A)^3}, \qquad \sin\alpha = \frac{1}{H}\sqrt{A(C-B)(2C-A)^3},$$

$$\cos\rho = \sqrt{\frac{C(A-B)(C-2A)}{(C-A)[3AC-B(A+C)]}}, \qquad \sin\rho = \sqrt{\frac{A(C-B)(2C-A)}{(C-A)[3AC-B(A+C)]}},$$

$$\chi = \sqrt{\frac{3AC-2B(A+C)}{3AC}}, \qquad \varkappa = \sqrt{\frac{(C-2A)(2C-A)}{3AC[3AC-B(A+C)]}},$$

$$v = -3AC\sqrt{\frac{Mgl}{H}}\sqrt{\frac{C-A}{3AC-B(A+C)}}, \qquad l = \sqrt{x_0^2 + z_0^2},$$

$$H = \sqrt{A(C-B)(2C-A)^3 + C(A-B)(C-2A)^3},$$

$$k = \sqrt{(C-2A)(2C-A)[3AC-B(A+C)]},$$

$$k' = (A+C)\sqrt{3(A-B)(C-B)}, \qquad N = 3AC\sqrt{3AC-B(A+C)}.$$

The conditions (2), as well as the solution (3), hold only for a body containing cavities filled with an ideal incompressible fluid.

If one looks closely at formulas (3), it is easy to see that they strongly resemble the formulas describing the regular precession of a kinetically asymmetric heavy rigid body (2), first found by Grioli (3). However, the motion (3) is not a regular precession. This is already clear from the fact that in the present case one of the basic kinematic conditions accompanying this motion is not satisfied:

$$p^2 + q^2 + r^2 = \omega^2 = \text{const}.$$

But in the present case there is the particular integral

$$Ap\cos\rho + Cr\sin\rho = \nu,$$

which can be interpreted in two ways: either as the scalar product of the vector (Ap, Bq, Cr) with the vector $(\cos\rho, 0, \sin\rho)$, or as the scalar product of the vector (p, q, r) with the vector $(A\cos\rho, 0, C\sin\rho)$, which remain constant in the course of the motion of the body. The latter circumstance of the motion will be a kinematic sign of regular precession if and only if the vector $(A\cos\rho, 0, C\sin\rho)$ coincides in direction with the normal to one of the circular sections of the inertia ellipsoid of the body and with the vector $(l\cos\alpha, 0, l\sin\alpha)$, determining the center of gravity of the body. If these conditions are required to be satisfied, then we obtain the following equalities, ensuring the existence of regular precessions for a body with cavities filled with fluid:

$$\cos\rho = \cos\alpha = 0, \qquad \sin\rho = \sin\alpha = 1, \qquad A\varkappa = \chi.$$

Under this restriction, conditions (2) are rewritten as follows:

$$A = B, \qquad C > 2A, \qquad x_0 = y_0 = 0, \qquad z_0 = l.$$

The solution describing regular precession in the present case is obtained in the following form:

$$p = \frac{\nu}{A}\chi\cos\sigma, \qquad q = \nu\varkappa\sin\sigma, \qquad r = \frac{\nu}{C},$$

$$\varkappa_1 = \frac{s}{\chi}\cos\sigma, \qquad \varkappa_2 = \frac{s}{A\varkappa}\sin\sigma, \qquad \varkappa_3 = \text{const},$$

where

$$s = \frac{C - 2A}{2C - A}, \qquad \text{const} = \frac{A + C}{2C - A},$$

and the function

$$\sigma = \frac{\nu}{N}kt + \sigma_0$$

is a linear function of time.

The direction cosines $\varkappa_1, \varkappa_2, \varkappa_3$, determining the orientation of the body' s axis of precession, as is not difficult to verify, coincide respectively with $\gamma, \gamma', \gamma''$. Thus the precessional motion under consideration takes place about the vertical axis. This is a new regular precession, differing from the known precession in the Lagrange case, in which, although $A = B$, nevertheless $2A > C$. In the case considered it is the only dynamically possible precession.

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3. G. Grioli, *Ann. Matem. pura ed appl.*, ser. IV, **26** (1947).

Note: Figure translations are in progress. See original paper for figures.