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Abstract

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GEOPHYSICS

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ON MAGNETOHYDRODYNAMIC EFFECTS IN THE OCEAN

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The seas and oceans of the Earth constitute a conducting medium ($\sigma \sim 10^{10}$ sec^{-1}), in which specific magnetohydrodynamic effects are possible in the Earth' s magnetic field. Owing to the relatively low conductivity of seawater $\left(\mathrm{Re}_m = \frac{4\pi\sigma}{c^2} vl \ll 1\right)$ in the terrestrial field $B \sim 0.5$ oersted, these effects are very weak and manifest themselves only in motions of sufficiently large scale; the possibility of the latter in the sea also makes it possible to regard it as a magnetohydrodynamic medium. The influence of a constant field B on hydrodynamic motions is, as a rule, negligibly small if these motions exist also for $B = 0$. Magnetohydrodynamic effects appear in the generation of currents and of an electromagnetic field accompanying hydrodynamic fields. Thus, the propagation of infrasound in the ocean is accompanied also by the propagation of the associated nondecaying electromagnetic field; ocean currents lead to local changes in the Earth' s magnetic field comparable with the amplitude of its diurnal variations, etc.*

1. For drift wind currents ([4]) it is easy to obtain an exact solution of the equations of magnetohydrodynamics. We consider the motion of a viscous, conducting, incompressible fluid in the half-space $z > 0$, stationary and homogeneous ($\partial/\partial t = \partial/\partial x = \partial/\partial y = 0$), under the action of the tangential frictional force T (at $z = 0$) of a wind blowing along the y-axis, and of the Coriolis force. From the equations $\operatorname{div} \mathbf{v} = 0,\ \operatorname{div} \mathbf{B} = 0,\ \mathbf{j} = \frac{c}{4\pi} \operatorname{rot} \mathbf{B},\ v_z|_{z=0} = 0$, it follows that $B_z = \text{const},\ v_z = 0,\ j_z = 0$. Using the boundary condition $\mathbf{v} = 0,\ d\mathbf{B}/dz = 0$ for $z = \infty$, we obtain the equations of magnetohydrodynamics (with allowance for the Coriolis force) for the complex velocity $w = v_x + iv_y$, magnetic field $\mathfrak{B} = B_x + iB_y$, and current density $j = j_x + ij_y$

$$d^2w/dz^2 - (b^2 \mp i\,2a^2)w = 0; \tag{1}$$

$$\frac{d\mathfrak{B}}{dz} = -\frac{4\pi\sigma}{c^2} B_z w; \quad (2)$$

$$j = -i\frac{\sigma}{c} B_z w, \quad (3)$$

where $a^2 = \Omega|\sin\varphi|/\nu$; $b^2 = B_z^2\sigma/\eta c^2$; $l = (a\sqrt{2})^{-1}$ is the characteristic (vertical) scale of the current in the absence of a magnetic field; b^{-1} is the characteristic scale associated with the action of electrodynamic and viscous forces; $M = b(a\sqrt{2})^{-1} = B_z l\sqrt{\sigma/\eta c^2}$ is the Hartmann number; Ω is the angular velocity of the Earth' s daily rotation; φ is the geographic latitude; η is the dynamic turbulent viscosity; $\nu = \eta/\rho$. The upper sign refers to the Northern, the lower to the Southern Hemisphere. The Hartmann number, characteriz—

* The existence of induced electric currents in the sea was predicted by Faraday ([1]); see also ([2]). For measurements of these currents and their influence on the Earth' s magnetic field see works ([2,3]) and the references cited therein.

···the effect of the magnetic field on the motion has an especially simple physical meaning in the case of a gas (if σ and η are understood as molecular quantities). Using the formulas of kinetic theory, $\eta \sim nm\bar{v}^2\tau$, $\sigma \sim ne^2\tau/m$, where τ is the mean free time, we obtain $M \sim l/r$, where $r = mc\bar{v}/eH$ is the radius of gyration in the magnetic field H.

For the electric field $E = \mathbf{j}/\sigma - [\mathbf{v}, \mathbf{B}]/c$, only the vertical component is nonzero,

$$E_z = \mathrm{Im}(w\mathfrak{B}^*)/c, \quad (4)$$

since $\mathcal{E} \equiv E_x + iE_y = j/\sigma + iB_z w/c = 0$ according to (3).

From (1)–(3) it is seen that $\mathfrak{B}$ and j are expressed in terms of the velocity w, satisfying equation (1), which, under the boundary conditions

$$dw/dz|_{z=0} = iT/\eta, \qquad w|_{z=\infty} = 0 \quad (5)$$

has the solution

$$w = iU\exp\{-\varepsilon' z \pm i(\alpha + \varepsilon'' z)\}, \quad (6)$$

where

$$U = T/\eta|\varepsilon|, \qquad 2\alpha = \mathrm{arcctg}\, M^2, \quad (7)$$

$$\varepsilon' + i\varepsilon'' = |\varepsilon|e^{i\alpha} = \sqrt{b^2 + i2a^2}, \qquad \varepsilon' > 0, \qquad \varepsilon'' > 0.$$

With the exception of the immediate vicinity of the equator (i.e., for $\varphi \lesssim \varphi_0 = \sigma B_z^2/2\Omega c^2\rho \sim 10^{-8}$ for $\sigma = 4 \cdot 10^{10}$ and $B_z = 0.1$ oersted at the equator), $M \ll 1$, and the effect of the magnetic field on the velocity is negligibly small. In this case (6) goes over into Ekman' s solution (1):

$$w = iU_0 \exp\{-az \pm i(\pi/4 + az)\}, \qquad U_0 = T/\eta a\sqrt{2}. \tag{8}$$

The velocity vector at the surface, equal in absolute value to U_0, is turned by 45° to the right (to the left in the Southern Hemisphere) relative to the wind and, with depth, rotates to the right (to the left in the Southern Hemisphere), decreasing exponentially in magnitude. The influence of the magnetic field on the current is formally possible at the equator in sufficiently deep sea in those regions where $B_z \neq 0$. According to (7), $\alpha \to 0$ as $\varphi \to 0$, $B_z \neq 0$, i.e., in the presence of a vertical component of the magnetic field at the equator, the velocity of the current at the surface is directed along the wind even in deep sea.* As is known, in ordinary hydrodynamics allowance for the finite depth of the sea leads to the same qualitative result.

Knowing the velocity w, we find the magnetic field from equation (2):

$$\mathfrak{B} = \mathfrak{B}_\infty = iH \exp\{-\varepsilon' z \pm i(2\alpha + \varepsilon'' z)\}, \qquad H = \frac{4\pi\sigma}{c^2}\frac{TB_z}{\eta|\varepsilon|^2}, \tag{9}$$

where $\mathfrak{B}_\infty$ is the limiting value of the field as $z \to \infty$, which we identify with the undisturbed value of the horizontal component of the Earth' s magnetic field.

For the magnetic field excited by the Ekman current (8), we obtain

$$\mathfrak{B} - \mathfrak{B}_\infty = iH_0 \exp\{-az \pm i(\pi/2 + az)\}, \tag{10}$$

where

$$H_0 = \frac{4\pi\sigma}{c^2} U_0 l B_z. \tag{11}$$

This solution represents the first term in the expansion of the exact solution (9) in the small Hartmann number.

Thus, in wind-driven currents there arises an additional horizontal magnetic–

* It is obvious, however, that as the horizontal component of the Coriolis force decreases, other non-electromagnetic forces, as well as inhomogeneity in the horizontal plane, begin to play a role, and the present treatment is inapplicable.

field (10). The field vector at the sea surface is turned by 90° to the right (in the Southern Hemisphere to the left) from the wind direction (and by 45° from the direction of the velocity) when $B_z > 0$. With depth the additional magnetic field rotates together with the velocity, decreasing exponentially in magnitude with the same period and damping constant. Depending on the wind strength, H_0 is of order 0.1—10 γ, which is a quantity comparable with the amplitude of the diurnal variations of the Earth' s magnetic field (thus, already at a wind speed of 6 m/sec ($U_0 \sim 20$ cm/sec) in middle latitudes $H_0 \sim 0.1\,\gamma$).

The estimate $\mathcal{H} \sim \frac{4\pi\sigma}{c^2} vlB$, similar to (11), holds in all problems where $\mathbf{v}$ and $\mathbf{H}$ vary essentially in only one of the directions, while v and l are the characteristic velocity and scale of the motion, and is applicable, of course, not only to drift currents. For rapid large-scale currents the additional magnetic field can reach hundreds of gammas.

The expression for the current density is obtained by substituting (9) into (3). The current is orthogonal to the velocity and at the surface makes an angle $\pi/2 - \alpha$ with the direction of the wind. For the Ekman current (8)

$$j = J_0 \exp\{-az \pm i(\pi/4 + az)\}, \qquad J_0 = \sigma B_z U_0/c. \tag{12}$$

For $U_0 \sim 20$ cm/sec, $J_0 \sim 10^{-8}$ A/cm^2.

Introducing the angle γ between the wind direction and the direction of the horizontal component of the Earth' s magnetic field, we write $\mathfrak{B}_\infty$ in the form $|\mathfrak{B}_\infty| e^{i\gamma}$. Using the smallness of the magnetic Reynolds number and of the Hartmann number, we obtain for the electric field, according to (4),

$$E_z = U_0 |\mathfrak{B}_\infty| c^{-1} \operatorname{Re} \exp\{-az + i(\pi/4 \mp \gamma + az)\}. \tag{13}$$

At a velocity $U_0 \sim 20$ cm/sec, $E_z \sim 10^{-5}$ V/m.

2. Magnetohydrodynamic effects also appear in large-scale low-frequency oscillations. Let $\sigma/\varepsilon \gg \omega \gg \omega_u = 4\pi\sigma u^2/c^2$ ($\sigma/\varepsilon \sim 10^8$ Hz, $\omega_u \sim 10^{-12}$ Hz for the sea). Oscillations corresponding to Alfvén waves and slow sound for $\sigma = \infty$ (electromagnetic waves in water for $B = 0$) are damped at the usual skin depth [5]. The dispersion law for these branches is $k^2/\omega^2 = i \cdot 4\pi\sigma/c^2\omega$. Modified sound (the fast magnetosonic wave for $\sigma = \infty$) corresponds to the dispersion law

$$\frac{k^2}{\omega^2} = \frac{1}{s^2}\left(1 - \frac{u^2}{s^2}\frac{\sin^2\theta}{1 + i\omega/\omega_s}\right), \qquad \omega_s = 4\pi\sigma\frac{s^2}{c^2}, \qquad \cos\theta = \frac{(\mathbf{kB})}{kB}, \tag{14}$$

where s is the speed of sound in the sea and u is the Alfvén speed $u = B/\sqrt{4\pi\rho}$ ($s = 1.5 \cdot 10^5$, $u^2/s^2 \sim 10^{-12}$); $\mathbf{k}$ is the wave vector of the plane wave; ω_s is the frequency at which the wavelength of sound and the wavelength of the electromagnetic wave in seawater become equal. For $\sigma = 4 \cdot 10^{10}$, $\omega_s/2\pi = 2$ Hz.

Thus the frequency range $\omega \sim \omega_s$ proves to be the most interesting, since at higher frequencies the hydromagnetic effects decrease (in [5] this limiting case of interest to us is absent).

The relation between the field amplitudes in an infrasonic wave ($\vec{\chi} = \mathbf{k}/k$, $\mathbf{h} = \mathbf{B}/B$):

$$\mathbf{H} = \frac{B}{\rho s^2}\frac{\sin\theta\,\vec{\eta}}{1 + i\omega/\omega_s}\,p', \qquad \mathbf{E} = -\frac{B}{\rho s c}\frac{\sin\theta\,\vec{\xi}}{1 + i\omega/\omega_s}\,p',$$

$$\mathbf{j} = i\frac{\omega c}{Bs}\frac{u^2}{s^2}\frac{\sin\theta\,\vec{\xi}}{1 + i\omega/\omega_s}\,p', \tag{15}$$

$$\mathbf{v} = (\vec{\chi}/\rho s)p', \qquad \vec{\eta}\sin\theta = [[\vec{\chi},\mathbf{h}],\vec{\chi}], \qquad \vec{\xi}\sin\theta = [\vec{\chi},\mathbf{h}].$$

For $\omega \sim \omega_s$, in the Gaussian system of units, $E \sim 10^{-15}p'$, $H \sim 10^{-10}p'$, where p' is the pressure amplitude in the sound wave. $\vec{\chi}$, $\vec{\eta}$, and $\vec{\xi}$ form an orthonormal basis associated with the wave.

In the "Alfvén wave" we have

$$\mathbf{H} = \sqrt{\frac{4\pi i}{\sigma\omega}}\,\vec{\xi} j, \qquad \mathbf{E} = \frac{\vec{\eta}}{\sigma}\,j, \qquad \mathbf{v} = -i\,\frac{B\cos\theta\,\vec{\xi}}{\rho\omega c}\,j, \qquad p' = 0, \qquad \mathbf{j} = \vec{\eta} j. \tag{16}$$

In the "slow magnetosonic wave"

$$\mathbf{H} = -\sqrt{\frac{4\pi i}{\sigma\omega}}\,\vec{\eta} j, \qquad \mathbf{E} = \frac{\vec{\xi}}{\sigma}\,j, \qquad \mathbf{v} = i\,\frac{B}{\rho\omega c}\left\{\frac{\sin\theta\,\vec{\chi}}{i\omega_s/\omega - 1} + \cos\theta\,\vec{\eta}\right\} j,$$

$$p' = B\sqrt{\frac{i}{4\pi\sigma\omega}\,\frac{\sin\theta}{1 + i\omega/\omega_s}}\,j, \qquad \mathbf{j} = j\vec{\xi}. \tag{17}$$

From (15) it follows for the attenuation coefficient of infrasound due to conductivity in a magnetic field (anisotropic induction absorption [6]):

$$\operatorname{Im} k = \frac{\omega^2}{2s^3}\frac{u^2}{s^2}\frac{c^2}{4\pi\sigma}\frac{\sin^2\theta}{1 + (\omega/\omega_s)^2}. \tag{18}$$

For $\omega \ll \omega_s$, this attenuation is of the same order as the attenuation due to molecular viscosity.

Peculiar phenomena should arise at the sea surface. When infrasound is incident on an interface (both from the air and from the sea), there arises not only reflected and refracted sound, but also decaying electromagnetic waves in the water and an electromagnetic wave above the sea (the sum of two waves with different polarizations). All these waves will also arise when an electromagnetic wave is incident on the interface (six "diverging" waves). From "Snell' s law" it follows that, at angles of incidence of the sound wave from the sea $\varphi^n > s/c \sim 10^{-5}$ (from the air, at $\varphi^n > \bar{s}/c \sim 10^{-6}$), the electromagnetic waves arising above the sea are surface waves with attenuation depth $(\operatorname{Im} k_\perp)^{-1} = \lambda/2\pi \sin\varphi^n$, where λ is the wavelength of the sound wave in seawater and $\bar{s}$ is the speed of sound in air. The electromagnetic wave propagates along the surface with a velocity equal to the horizontal component of the sound velocity.

When sound with amplitude p'_n is incident, the field amplitudes in the electromagnetic wave are $E \sim 10^{-15} p'_n$, $H \sim 10^{-10} p'_n$.

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Note: Figure translations are in progress. See original paper for figures.