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Abstract

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THEOREMS OF SYLOW TYPE

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§ 1. A cycle of papers ([1−10]) contains numerous analogues and generalizations of Sylow' s theorem on the embedding of subgroups in the theory of finite groups. In the article ([13]) (see also ([14])) we proved two theorems on the embedding of subgroups; moreover, theorem 1 generalizes the known results of G. Wielandt ([1]) and Tibiletti ([9]).

In the present note we give a further series of theorems obtained by us on the embedding of subgroups, connected with the cyclicity of certain Sylow subgroups of the given group. From theorem 1 of the present note there follows the result of K. Honda ([10]), as well as theorem 2 proved by us earlier ([13]).

§ 2. We shall denote by Π some nonempty set of primes; by $\Pi(d)$ the totality of all prime divisors of the number d; and by $\mathfrak{G}$ a finite group of order $(\mathfrak{G})$. Following S. A. Chunikhin ([11]), every divisor d of the number $(\mathfrak{G})$ such that $\Pi(d) \subseteq \Pi$ will be called a Π-divisor of $(\mathfrak{G})$.

Definition 1. We shall call a subgroup of the group $\mathfrak{G}$ a Π-subgroup if its order is a Π-divisor of $(\mathfrak{G})$.

It is obvious that the identity subgroup of the group $\mathfrak{G}$ also belongs to the class of Π-subgroups.

A subgroup whose order is equal to the greatest Π-divisor of $(\mathfrak{G})$ will be called, following Wielandt ([2]), a Π-Hall subgroup of the group $\mathfrak{G}$.

Definition 2. We shall say that for the group $\mathfrak{G}$ the strong Π-Sylow theorem holds if, for every Π-subgroup $\mathfrak{H}$ of the group $\mathfrak{G}$ and for every subgroup $\mathfrak{M}$ whose order divides the order of $\mathfrak{H}$, there exists an element $G \in \mathfrak{G}$ such that

$$\mathfrak{M}^G = G^{-1}\mathfrak{M}G \subseteq \mathfrak{H}.$$

Definition 3. If there exists a Π-Hall subgroup $\mathfrak{G}_\Pi$, and for every subgroup $\mathfrak{M}$ whose order divides the order of $\mathfrak{G}_\Pi$ there is an embedding

$$\mathfrak{M}^G \subseteq \mathfrak{G}_\Pi, \quad G \in \mathfrak{G},$$

then we shall say that for $\mathfrak{G}$ the Π-Sylow theorem holds ([2]).

In the present paper we also use the concept of a $\Pi\Delta$-group, introduced by us in ([12]).

§ 3. **Theorem 1.** *Let to each prime divisor from Π there correspond a cyclic Sylow subgroup of the group $\mathfrak{G}$. Then for $\mathfrak{G}$ the strong Π-Sylow theorem holds.*

Theorem 2. *Let Π, $\sigma = \{p\}$, and τ be sets of primes such that $\sigma \cap \tau$ is empty and $\Pi = \sigma \cup \tau$, and suppose that to the prime number p there corresponds a cyclic Sylow subgroup of the group $\mathfrak{G}$.*

Let $\mathfrak{G}$ have a Π-subgroup

$$\mathfrak{H} = \mathfrak{H}_p \times \mathfrak{G}_\tau,$$

where $\mathfrak{H}_p$ is a p-Sylow subgroup of the group $\mathfrak{H}$, and $\mathfrak{G}_\tau$ is a τ-Hall subgroup of the group $\mathfrak{G}$.

If for $\mathfrak{G}$ the τ-Sylow theorem holds, then for any subgroup $\mathfrak{M}$ whose order divides the order of $\mathfrak{H}$, there exists an element $G \in \mathfrak{G}$ such that

$$\mathfrak{M}^G \subseteq \mathfrak{H}.$$

Theorem 3. *Let Π, σ, and τ be such (empty or nonempty) sets of prime numbers that $\sigma \cap \tau$ is empty and $\Pi = \sigma \cup \tau$, and suppose that to each prime number from σ there corresponds a cyclic Sylow subgroup of the group $\mathfrak{G}$.*

Let G have a Π-subgroup $H = H_\sigma \times G_\tau$, where H_σ is a σ-Hall subgroup of H, and G_τ is a τ-Hall subgroup of G.

If the τ-Sylow theorem holds for G, then for any σ-solvable subgroup M whose order divides the order of H, there exists an element $G \in G$ such that $M^G \subseteq H$.

Theorem 4. Let Π, σ, and τ be such (empty or nonempty) sets of primes that $\sigma \cap \tau$ is empty, $\Pi = \sigma \cup \tau$, and every element of σ is smaller than every element of τ; moreover, to each prime number in σ there corresponds a cyclic Sylow subgroup of the group G.

Let G have a Π-subgroup $H = H_\sigma \times G_\tau$, where H_σ is a σ-Hall subgroup of H, and G_τ is a τ-Hall subgroup of G.

If the τ-Sylow theorem holds for G and M is any subgroup whose order divides the order of H, then there exists an element $G \in G$ such that $M^G \subseteq H$.

Theorem 5. Let Π, σ, and τ be such (empty or nonempty) sets of primes that $\sigma \cap \tau$ is empty and $\Pi = \sigma \cup \tau$, and to each prime number in σ there corresponds a cyclic Sylow subgroup of the group G.

Let G have a Π-subgroup $H = H_\sigma \times G_\tau$, where H_σ is a σ-Hall subgroup of H, and G_τ is a τ-Hall subgroup of G.

If the τ-Sylow theorem holds for G, then for any $\sigma\Delta$-subgroup M whose order divides the order of H, there exists an element $G \in G$ such that $M^G \subseteq H$.

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Note: Figure translations are in progress. See original paper for figures.