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Abstract

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HYDRODYNAMIC THEORY OF CLIMATE AND LONG-RANGE WEATHER FORECASTING

In paper ([1]) we reduced the problem of the hydrodynamic long-range forecast of meteorological elements to the solution of a system of differential equations describing, along with nonperiodic processes (dependent on the initial data), also processes having an annual period (climatic ones). The basic equations were: the equation of heat influx, the equation of vortex transport, and a generalized balance equation. In the present paper we set out a method for solving these equations.

The equation of heat influx (equation (9) from ([1])) has the form:

$$\frac{\partial T'}{\partial t}+\frac{v_\theta}{a_0}\frac{\partial T'}{\partial\theta}+\frac{v_\lambda}{a_0\sin\theta}\frac{\partial T'}{\partial\lambda}-\frac{\gamma_a}{g}\frac{\partial\Phi}{\partial t}+\frac{RT_{\text{cp}}}{g}(\gamma_a-\gamma)\frac{W}{\xi}=$$

$$=\frac{1}{N}\left(\frac{dT}{dE}\right)_{\text{cp}}\left[\frac{2}{m^2-1}\frac{\partial^2 E}{\partial\zeta^2}-2E+e^{-\zeta}\int_0^\zeta Ee^{\zeta}d\zeta-e^{\zeta}\int_Z^\zeta Ee^{-\zeta}d\zeta+\right.$$

$$\left.+e^{\zeta-Z}(E)_{\zeta=Z}+\beta S_0e^{-\beta\zeta}\right]=F \qquad (1)$$

(here and below all notation is as in ([1])). The corresponding boundary condition (formula (12) from ([1])) is written in the form:

$$\frac{2}{m^2-1}\frac{\partial E}{\partial\zeta}+\lambda^*\frac{\partial T^*}{\partial z}=e^{-Z}\int_0^Z Ee^{\zeta}d\zeta-r\sigma T_Z^4+S_0e^{-\beta Z}\quad \text{for } \zeta=Z. \qquad (2)$$

Here T^* satisfies the equation (formula (13) from ([1]))

$$c^*\rho^*\frac{\partial T^*}{\partial t}=\frac{\partial}{\partial z}\left(\lambda^*\frac{\partial T^*}{\partial z}\right) \qquad (3)$$

and the boundary conditions

$$(T^*)_{z=0} = (T)_{z=0}, \qquad \left(\frac{\partial T^*}{\partial t}\right)_{z=-\infty} = 0. \tag{4}$$

The right-hand side of (1) contains the function T (or E) itself, and not its derivatives with respect to θ, λ, t. The separate terms entering the right-hand side of (1) are an order of magnitude larger than the terms of the left-hand side of (1). Indeed, if we take, according to ([2]), $\alpha_2 \rho_w \approx 4.5 \cdot 10^{-5}$ cm^{-1}, and take from Table 1 of paper ([2])

$$[dE/dT]_{\rm cp} \approx {}^1/_{300} \text{ cal/deg} \cdot \text{cm}^2 \cdot \text{min},$$

then we obtain

$$N = \left(\frac{c_p \rho}{\alpha_2 \rho_w} \frac{dT}{dE}\right)_{\rm cp} = 1.2 \cdot 10^5 \text{ sec.};$$

then, for the mean value $E \approx 0.190$ cal/cm$^2 \cdot$ min, we obtain the contribution of the term containing $2E$, in the form

$$\frac{1}{N}\left(\frac{dT}{dE}\right)_{\rm cp} 2E = 0.95 \cdot 10^{-3} \text{ deg/sec.}$$

At the same time the term $\partial T'/\partial t$, standing in the left-hand side of (1), has the characteristic value 10^{-4} deg/sec. The computational difficulty connected with this difference in orders of magnitude is easily removed by separating out, in the right-hand side of (1), the standard temperature $\widetilde{T}(z)$. If we assume $E(T) = E(\widetilde{T}) + (dE/dT)_{\rm cp} T'$, then instead of the right-hand side of (1) we may write

$$F = \frac{1}{N}\left[\frac{2}{m^2-1}\frac{\partial^2 T'}{\partial \zeta^2} - 2T' + e^{-\zeta}\int_0^\zeta T' e^{\zeta} d\zeta - e^{\zeta}\int_Z^\zeta T' e^{-\zeta} d\zeta + \right.$$

$$\left. + \mu e^{\zeta - Z} T'_Z \left(\frac{dT}{dE}\right)_{\rm cp} + \beta S'_0 e^{-\beta\zeta}\left(\frac{dT}{dE}\right)_{\rm cp}\right], \tag{5}$$

where $S'_0(\theta, \lambda, t) = S_0(\theta, \lambda, t) - \widetilde{S}_0$, i.e., the deviation of S_0 from its value $\widetilde{S}_0$ corresponding to the standard temperature distribution.

Similarly, we write the boundary condition (2) in the form

$$\frac{2}{m^2-1}\frac{\partial T'}{\partial \zeta} + \left(\frac{dT}{dE}\right)_{\text{cp}} \lambda^* \frac{\partial T^*}{\partial z} = e^{-Z}\int_0^Z T' e^{\zeta} d\zeta - \mu \left(\frac{dT}{dE}\right)_{\text{cp}} T'_z +$$

$$+ \left(\frac{dT}{dE}\right)_{\text{cp}} S'_0 e^{-\beta Z} \quad \text{for } \zeta = Z^*. \tag{6}$$

It is expedient also to separate out the "standard" temperature of the Earth $\widehat{T}^*(z, \theta, \lambda, t)$, which, unlike the standard temperature $\widetilde{T}(z)$, must be regarded as a function of time (and of the coordinates θ and λ) possessing an annual period; it may be determined in advance from (3) (p. 18).

Keeping in mind the solution of the problem by time steps, we replace in equations (1) and (3) the time derivatives by finite differences between the values of the functions at the moments $t + \delta t$ and t, writing the terms that do not contain differentiation with respect to t at the moment $t + \delta t$ (implicit derivatives). Equation (3) becomes the relation

$$\frac{\partial^2 T^{*\,t+\delta t}}{\partial z^2} - \frac{T^{*\,t+\delta t}}{k^* \delta t} = -\frac{T^{*\,t}}{k^* \delta t} \tag{7}$$

$$\left(T^* = T - \widehat{T}^*;\ k^* = \lambda^*/c^*\rho^*; \quad \text{approximately one may take } \partial\lambda^*/\partial z \approx 0\right),$$

which, using conditions (4), makes it possible to determine the quantity $\lambda^*(\partial T^*/\partial z)_{z=0}$:

$$\lambda^* \left(\frac{\partial T^{*\,t+\delta t}}{\partial z}\right)_{z=0} = \frac{\lambda^*}{\sqrt{k^* \delta t}} \left(T^{*\,t+\delta t}\right)_{z=0} - \frac{\lambda^*}{k^* \delta t} \int_{-\infty}^{0} e^{z'/\sqrt{k^* \delta t}} T^{*\,t}\, dz'. \tag{8}$$

Then an approximate expression for the quantity $\lambda^*(\partial T^*/\partial z)_{z=0}$ entering into (6) may, with allowance for (4), be written in the form

$$\lambda^* \left(\frac{\partial T^{*\,t+\delta t}}{\partial z}\right)_{z=0} = \frac{\lambda^*}{\sqrt{k^* \delta t}} \left(T'^{\,t+\delta t} - T'^{\,t}\right)_{z=0} + Q^{t+\delta t}, \tag{9}$$

where

$$Q^{t+\delta t} = \lambda^* \left(\partial \widehat{T}^{*\,t+\delta t}/\partial z\right)_{z=0}$$

is a known quantity, substantially dependent on θ and λ and quasi-stationary (more precisely, possessing an annual cycle).

We divide the interval $0 \le \xi \le 1$ into subintervals with endpoints at the points $0, \xi_1, \xi_2, \dots, \xi_{n-1}, \xi_n$ ($\xi_n = 1$) and write (1), replacing in it the derivatives with respect to ζ by finite differences, and the integrals by sums according to the trapezoidal formula; in the right-hand side of (1) we shall take the value of E (or T') at the points $0, \xi_1, \xi_2, \dots, \xi_n$ (the corresponding values of ζ will be denoted by $0, \xi_1, \xi_2, \dots, \xi_n$ ($\xi_n = Z$)). We then obtain n equalities:

$$f_{\xi_k}^{t+\delta t} + \frac{RT_{\text{cp}}}{g}\,\delta t\,(\gamma_a - \gamma)_{\xi_k}^{t}\frac{W_{\xi_k}^{t+\delta t}}{\xi_k} = \frac{\delta t}{N}\left[\sum_{i=1}^{n} a_{ik} T'^{\,t+\delta t}_{\xi_i} + \right.$$

$$\left. + \left(\frac{dT}{dE}\right)_{\text{cp}} \beta S_0'^{\,t+\delta t} e^{-\beta\xi_k}\right] \quad (\text{for } 1 \le k \le n-1), \tag{10}$$

$$f_1^{t+\delta t} = \frac{\delta t}{N}\sum_{i=1}^{n} a_{in} T'^{\,t+\delta t}_{\xi_i} + \frac{\delta t}{N}\left(\frac{dT}{dE}\right)_{\text{cp}}\left\{(\beta + b) S_0'^{\,t+\delta t} e^{-\beta Z} - \right.$$

$$\left. - b\left[Q^{t+\delta t} + \frac{\lambda^*}{\sqrt{k^*\delta t}}\left(T_1'^{\,t+\delta t} - T_1'^{\,t}\right)\right]\right\}. \tag{11}$$

* As $\widehat{T}(z)$ we may take, for example, the expression $\widetilde{T}$ corresponding to $E_0^0(\zeta)$

from work ([2]); in this case it was

$$\left(\lambda^* \frac{\partial T^*}{\partial z}\right)_{z=0} = 0.$$

(for $k = n,\ \xi_n = 1$); in this case

$$t_{\xi_k}^{t+\delta t} = T'^{\,t+\delta t}_{\xi_k} - T'^{\,t}_{\xi_k} - \frac{\gamma_a}{g}\left(\Phi_{\xi_k}^{t+\delta t} - \Phi_{\xi_k}^{t}\right) + \frac{\delta t}{a_0}\left(v_\theta \frac{\partial T'}{\partial \theta} + \frac{v_\lambda}{\sin\theta}\frac{\partial T'}{\partial \lambda}\right)_{\xi_k}^{t+\delta t/2} {}^*.$$

The coefficients a_{ik} and b are easily computed. In deriving (11), the boundary conditions (6) and (9) were used.

Putting $a_{ik} = b = \beta = 0$, we arrive at the usual system for adiabatic forecasts. The presence of right-hand sides in (10) and (11) introduces something fundamentally new into the treatment of the stratosphere and of the layers adjacent to the Earth' s surface (as compared with the adiabatic formulation of the problem).

Let us give, as an example, the case of division into intervals $\xi_1 = 0.1;\ \xi_2 = 0.2;\ \dots;\ \xi_{n-2} = 0.9;\ \xi_{n-1} = 0.95;\ \xi_n = 1$. A simple calculation with the parameters chosen by us $\left(Z = 12.6;\ \dfrac{2}{m^2-1} = 1;\ \zeta = Z\xi^{1.852}\right)$ gives

$$\sum_{i=1}^{n} a_{i1} {T'_{\xi_i}}^{t+\delta t} = (-26.139\, T'_{0.1} + 7.096\, T'_{0.2} + 0.255\, T'_{0.3} + 0.136\, T'_{0.4} +$$

$$+0.055\, T'_{0.5} + 0.017\, T'_{0.6} + 0.004\, T'_{0.7} + 0.001\, T'_{0.8})^{t+\delta t}\,. \qquad (12)$$

For $\delta t = 3$ hours we obtain $\delta t/N = 0.1$. This means that for $\xi_k = \xi_1 = 0.1$ one should rather adopt the relation

$$\sum_{i=1}^{n} a_{i1} {T'_{\xi_i}}^{t+\delta t} + \left(\frac{dT}{dE}\right)_{\mathrm{cp}} \beta {S'_0}^{t+\delta t} e^{-\beta \xi_1} \approx 0$$

(from (10)), and not the condition of adiabaticity.

In the middle troposphere the coefficients a_{ik} will be different. For example, for $\xi_k = 0.7$ we have

$$\sum_{i=1}^{n} a_{i7} {T'_{\xi_i}}^{t+\delta t} = (0.001\, T'_{0.1} + 0.002\, T'_{0.2} + 0.006\, T'_{0.3} + 0.022\, T'_{0.4} +$$

$$+0.092\, T'_{0.5} + 0.835\, T'_{0.6} - 1.858\, T'_{0.7} + 0.808\, T'_{0.8} + 0.078\, T'_{0.9} +$$

$$+0.009\, T'_{0.95} + 0.011\, T'_1)^{t+\delta t}\,. \qquad (13)$$

We see that for $\xi_k = \xi_7 = 0.7$ relation (10) may approximately be regarded as adiabatic.

At the Earth itself the nonadiabatic influences again appear, but they now have quite a different character than in the stratosphere (4).

For $\zeta = Z$ ($\xi_n = 1$) the sum from the right-hand side of (11) takes the form

$$\sum_{i=1}^{n} a_{in} {T'_i}^{t+\delta t} = (0.003\, T'_{0.6} + 0.019\, T'_{0.7} + 0.144\, T'_{0.8} + 0.420\, T'_{0.9} +$$

$$+2.762\, T'_{0.95} - 3.654\, T'_1)^{t+\delta t}\,. \qquad (14)$$

These terms, generally speaking, are not the principal ones. Nonadiabatic influences manifest themselves through the underlying surface. The term containing Q describes here the moderating influence of the underlying surface on the change in temperature. The dependence of the transformation of air on the character of the underlying surface (ocean, land) is described by the term ∗∗ with λ^*.

* Here and below, nonlinear terms are written at the time $t + \delta t/2$.

** This term must be combined with the term ${T'_1}^{t+\delta t} - {T'_1}^{t}$ standing on the left in (11); for $\delta t = 10^4$ sec., $\delta t/N = 0.1$ will be

$$\left(1 + \frac{31.53\,\lambda^*}{\sqrt{k^*}}\right)\left({T'_1}^{t+\delta t} - {T'_1}^{t}\right);$$

for $\lambda^*/\sqrt{k^*} = 0.04$ cal $\cdot$ sec$^{-1/2}$/cm^2 $\cdot$ deg (for land)

$$1 + \frac{31.53\lambda^*}{\sqrt{k^*}} = 2.26,$$

i.e., we obtain

The new scheme for solving the forecasting problem that we propose is as follows. Equations (10) (the equation of heat influx) will serve to determine $W^{t+\delta t}_{\xi_k}$ (with replacement in these equations of T' by Φ according to formula (3) from (1)); to determine T'_{ξ_n}, equation (11) is used.

We rewrite the vorticity-transport equation (formula (7) from (1)) in the form

$$\Delta\psi^{t+\delta t}_{\xi_k} + \delta t\,\frac{2\omega a_0^2\cos\theta}{\xi_{k+1} - \xi_{k-1}}\left(W_{\xi_{k+1}} - W_{\xi_{k-1}}\right)^{t+\delta t} = \Delta\psi^{t}_{\xi_k} + \frac{\delta t}{\tau_{\text{cp}}\sin\theta}\left(\Phi, T'\right)^{t+\delta t/2}_{\xi_k} +$$

$$+\delta t\left\{\frac{(W\Delta\psi)_{\xi_{k+1}} - (W\Delta\psi)_{\xi_{k-1}}}{\xi_{k+1} - \xi_{k-1}} - 2(\Delta\psi)_{\xi_k}\frac{(\Delta\varphi)_{\xi_k}}{a_0^2} + \right.$$

$$+\frac{a_0}{\xi_{k+1} - \xi_{k-1}}\left(v_\lambda\frac{\partial W}{\partial\theta} - \frac{v_\theta}{\sin\theta}\frac{\partial W}{\partial\lambda}\right)\Bigg|_{\xi=\xi_{k-1}}^{\xi=\xi_{k+1}} - \left(\frac{v_\lambda}{a_0}\frac{\partial\Delta\varphi}{\partial\theta} - \frac{v_\theta}{a_0\sin\theta}\frac{\partial\varphi}{\partial\lambda}\right)_{\xi_k}$$

$$\left. - \left[\frac{v_\lambda}{a_0\sin\theta}\frac{\partial\Delta\psi}{\partial\lambda} + \frac{v_\theta}{a_0}\frac{\partial}{\partial\theta}\left(\Delta\psi + 2\omega a_0^2\cos\theta\right)\right]_{\xi_k}\right\}^{t+\delta t/2}. \qquad (15)$$

and substitute into it the values $W^{t+\delta t}$ determined by means of (10) and (11). We obtain a system of equations containing $\psi^{t+\delta t}$ and $\Phi^{t+\delta t}$.

The balance equation (formula (8) from (1)) gives a second system of equations for the functions $\psi^{t+\delta t}$ and $\Phi^{t+\delta t}$. This system has the form:

$$2\omega\cos\theta\,\Delta\psi^{t+\delta t}_{\xi_k} - \Delta\Phi^{t+\delta t}_{\xi_k} - \frac{a_0^2\left(W_{\xi_{k+1}} - W_{\xi_{k-1}}\right)^{t+\delta t}}{\delta t\left(\xi_{k+1} - \xi_{k-1}\right)} =$$

$$= \left\{ 2\left(\omega \sin\theta\, a_0 v_\lambda - \frac{1}{a_0}\frac{\partial v_\lambda}{\partial \theta}\Delta\psi\right)_{\xi_k} + 2\left[\frac{v_\theta}{a_0}\frac{\partial \Delta\varphi}{\partial\theta} + \frac{v_\lambda}{a_0 \sin\theta}\frac{\partial\Delta\varphi}{\partial\lambda} + \right.\right.$$

$$\left. + \frac{(\Delta\varphi)^2}{a_0^2} + \left(\frac{\partial v_\theta}{\partial\theta}\right)^2 + \left(\frac{\partial v_\lambda}{\partial\theta}\right)^2 + \frac{v_\theta^2 + v_\lambda^2}{2} - \frac{\partial v_\theta}{\partial\theta}\frac{\Delta\varphi}{a_0}\right]_{\xi_k}$$

$$\left. - \frac{a_0}{\xi_{k+1} - \xi_{k-1}}\left(\frac{W\Delta\varphi}{a_0} + v_\theta\frac{\partial W}{\partial\theta} + \frac{v_\lambda}{\sin\theta}\frac{\partial W}{\partial\lambda}\right)\Bigg|_{\xi=\xi_{k-1}}^{\xi=\xi_{k+1}} \right\}^{t+\delta t/2} - \frac{\Delta\Phi_{\xi_k}^t}{\delta t}. \tag{16}$$

These two systems contain on the right-hand sides $v_\theta, v_\lambda, \Delta\varphi$, and $\Delta\psi$. At the initial moment v_θ, v_λ and, hence, $\Delta\varphi, \Delta\psi$ are known. But how are they to be found subsequently? Let us note that, from the equation of continuity,

$$\Delta\varphi_{\xi_k}^{t+\delta t} = \frac{a_0^2}{\xi_{k+1} - \xi_{k-1}}\left(W_{\xi_{k+1}} - W_{\xi_{k-1}}\right)^{t+\delta t}. \tag{17}$$

Thus, by (17) we can find $\Delta\varphi^{t+\delta t}$; by inverting the Laplace operator, we find $\varphi^{t+\delta t}$. Further, from (15), by inverting the Laplace operator, we find $\psi^{t+\delta t}$. Now, by formulas (5) from (1), we can find $v_\theta^{t+\delta t}$ and $v_\lambda^{t+\delta t}$ and, in this way, close the time step.

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some smoothing of the temperature change that occurs due to advection.

Over the sea, $\lambda^*/\sqrt{k^*}$ is approximately 20 times greater than over land. If one takes $\lambda^*/\sqrt{k^*} \approx 1$, then the smoothing factor proves equal to 1/32.53; the effect of advection will practically be absent—the well-known fact of the invariability of the temperature of the sea surface. In the CGS system, $b(dT/dE)_{\text{cp}} \sim 3.153\cdot 10^4$.

Note: Figure translations are in progress. See original paper for figures.