ON THE STRUCTURAL NOISE IMMUNITY OF ONE CLASS OF CLOSED DYNAMIC SYSTEMS

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

CYBERNETICS AND CONTROL THEORY

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ON THE STRUCTURAL NOISE IMMUNITY OF ONE CLASS OF CLOSED DYNAMIC SYSTEMS

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A closed dynamic system is considered, particular cases of which are automatic-control systems and other systems with negative feedback.

The system consists of N elements with transfer functions $K_i R_i(P)/D_i(P)$. Of the total number of elements, $\alpha + m$ are subject to the action of disturbances; these elements are located at different places in the closed chain. For simplicity, let us assume that α elements subject to the action of disturbances are concentrated in one place, while the remaining m are located in another place of the chain, and that between α and m there are β elements not subject to the action of disturbances. This arrangement of the elements in the dynamic chain has been chosen to simplify the exposition. The conclusions that will be obtained can readily be used for any arrangement of the elements. The only condition imposed on the arrangement of the elements is the following. Taking as the beginning, or input, of the dynamic chain the place where the useful signal is applied (we assume that at the input there is a useful signal containing no disturbances), we shall suppose that the first ν elements are not subject to the action of disturbances, where ν is any number different from zero. Since a disturbance may be applied not only to the input of an element, we shall assume that the transfer function from the output of the given element to the point of application of the disturbance is different from the transfer function of this element, and denote it by $K_i' R_i'(P)/D_i'(P)$.

Fig. 1

Figure 1 presents the block diagram of the case under consideration.

A single restriction is imposed on the nature of the acting disturbances, namely that they and all their derivatives must be bounded in modulus,

$$\left|f_i^{(i)}\right|_{i=0,1,\ldots,n-1} < M;$$

otherwise the disturbances may be arbitrary, in particular random functions of time.

Below, the following property of the class of structures under consideration is proved, which we have called structural noise immunity: the accuracy of reproduction of the useful signal x_{in}, applied to the input of the system, will be the greater the larger are the gain coefficients of the elements that are not sub-

respectively, are not subjected externally to the action of disturbances; the disturbances will be suppressed by the gain coefficients of the elements located in the dynamic chain before the elements subjected to the action of disturbances.

For the proof, let us find the transfer function of the system represented in Fig. 1.

For elements not subjected to the action of disturbances, one may write:

$$x_{i+1} = \frac{K_i R_i(P)}{D_i(P)} x_i. \tag{1}$$

For elements subjected to the action of disturbances:

$$x_{j+1} = \frac{K_j R_j(P)}{D_j(P)} x_j + \frac{K_j' R_j'(P)}{D_j'(P)} f_j. \tag{2}$$

On the basis of Fig. 1, carrying out the calculations, we obtain the following expression for the output quantity:

$$\begin{aligned}
\left[\prod_{i=1}^{\nu+\alpha+\beta+m} \frac{K_i R_i(P)}{D_i(P)} \frac{K_n R_n(P)}{D_n(P)} + 1\right] x_{\text{out}} &= \prod_{i=1}^{\nu+\alpha+\beta+m} \frac{K_i R_i(P)}{D_i(P)} \frac{K_n R_n(P)}{D_n(P)} x_{\text{set}} \\
&+ \frac{K_n R_n(P)}{D_n(P)} \prod_{i=\alpha+1}^{\beta+m} \frac{K_i R_i(P)}{D_i(P)} \sum_{j=2}^{\alpha+j} \prod_{\rho+j}^{\alpha} \frac{K_{\nu+\rho} R_{\nu+\rho}(P)}{D_{\nu+\rho}(P)} \frac{K'_{\nu+j-1} R'_{\nu+j-1}(P)}{D'_{\nu+j-1}} \\
&+ \frac{K_n R_n(P)}{D_n(P)} \sum_{j=2}^{m+1} \prod_{\rho=j}^{m} \frac{K_{\nu+\alpha+\beta+j}}{D_{\nu+\alpha+\beta+j}(P)} \frac{K'_{\nu+\alpha+\beta+j-1} R'_{\nu+\alpha+\beta+j-1}}{D'_{\nu+\alpha+\beta+j-1}} f_{\alpha+j-1} \\
&+ \frac{K_n' R_n'(P)}{D_n'(P)} f_n.
\end{aligned} \tag{3}$$

Suppose that the gain coefficients of the elements which are not subjected to the direct action of disturbances can be chosen sufficiently large. Then, dividing all of equation (3) by $\prod_{i=1}^{\nu} K_i \prod_{j=\nu+\alpha+1}^{\beta}$ and denoting $1/K_i = m$, after elementary transformations we obtain:

$$\Bigg[m^{\nu+\beta} \prod_{i=1}^{\nu+\alpha+\beta+m} D_i(P) \prod_{j=\nu+1}^{\alpha} D'_j(P) \prod_{\rho=\nu+\alpha+\beta+1}^{m} D'_\rho(P) D_n(P) + K_n R_n(P) D'_n(P) \prod_{i=\nu+1}^{\alpha} K_i R_i(P) \prod_{j=\nu+1}^{\alpha} D'_j(P)$$

$$\times \prod_{j=\nu+\alpha+\beta+1}^{m} K_j R_j(P) \prod_{\rho=\nu+\alpha+\beta+1}^{m} D'_\rho(P) \Bigg] x_{\text{out}}$$

$$= K_n R_n(P) D'_n(P) \prod_{i=\nu+1}^{\alpha} K_i R_i(P) \prod_{j=\nu+\alpha+\beta+1}^{\alpha} K_j R_j($$

$$\times \prod_{j=\nu+1}^{\alpha} D'_j(P) \prod_{\rho=\nu+\alpha+\beta+1}^{m} D'_\rho(P) x_{\text{set}}$$

$$+ m^{\nu} K_n R_n(P) \prod_{i=\alpha+1}^{\beta+m} R_i(P)$$

$$\times \sum_{\rho=1}^{\alpha} \prod K_{\nu+\rho} R_{\nu+\rho}(P) K'_{\nu+j-1} R'_{\nu+j-1}(P) \prod_{i=1}^{\nu+\alpha} D_i(P$$

$$\times \{ D_{\nu+1}(P) + D_{\nu+2}(P) D_{\nu+1}(P) + \dots + \prod_{v=1}^{\alpha+\beta} D_i(P)\}$$

$$+ m^{\nu+\beta} K_n R_n(P) \prod_{i=1}^{\nu+\alpha+\beta} D_i(P) \big[D_{\nu+\alpha+\beta}(P) + D_\nu$$

$$\dots + \prod_{j=1}^{m} D_{\nu+\alpha+\beta+j}(P) \sum_{j=2}^{m+1} \prod_{\rho=1}^{m} K_{\nu+\alpha+\beta+\rho} R_{\nu+\alpha+\beta+j}(P) \prod_{i=\nu+1}^{\alpha} D'_i(P) \times$$
$$\times \prod_{j=\nu+\alpha+\beta+l}^{\nu+\alpha+\beta+m} D'(P) K'_{\nu+\alpha+\beta+j-1} R'_{\nu+\alpha+\beta+j-1}(P) f_{\alpha+j-1} + \tag{4}$$
$$+ m^{\nu+\beta} \prod_{i=1}^{\nu+\alpha+\beta+m} D_i(P) \prod_{i=\nu+1}^{\alpha} D'_i(P) \prod_{j=\nu+\alpha+\beta+1}^{m} D'_i(P) K'_n R'_n(P) f_n.$$

From equation (4) it is clear that

$$\lim_{m\to 0} x \quad = x \ . \tag{5}$$

Thus it has been proved that:

2. For the class of structures under consideration, disturbances can be suppressed by increasing the gain coefficients of the elements on which the disturbances do not act directly.

3. The disturbances will be suppressed the more strongly, the farther from the beginning the element on which the disturbances act is located, the more elements there are before this element on which disturbances do not act directly, and the larger the gain coefficients of the latter. In the general case, the equation for the output quantity in the class of structures under consideration can be written in the following form:

$$\left[m^{\nu+\beta+\gamma+\cdots} F_{N_0}(P) + R_0(P)\right] x \quad = R_0(P)x \quad + m^{\nu} \sum_i F_{\nu+i}(P) f_i +$$

$$+ m^{\nu+\beta} \sum_j F_{\nu+\beta+j}(P) f_j + \ldots + m^{\nu+\beta+\gamma+\cdots} F_n(P) f_n, \tag{6}$$

and, for $m \to 0$, $x \quad = x$.

For implementation it is necessary that the system remain stable as $m \to 0$, or, equivalently, as $K_i \to \infty$. This condition will be satisfied if the structures under consideration belong to the class of structures stable under an unbounded increase of the gain coefficients (1).

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CITED LITERATURE

1. M. V. Meerov, *Synthesis of Structures of High-Accuracy Automatic Control Systems*, 1959.

Note: Figure translations are in progress. See original paper for figures.

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