ON THE PROPERTIES OF THE SOLUTION OF THE PROBLEM OF A POINT EXPLOSION IN COMPRESSIBLE MEDIA

1961

SovietRxiv

Abstract

Full Text

HYDROMECHANICS

N. N. KOCHINA and N. S. MEL' NIKOVA

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(Presented by Academician L. I. Sedov, 15 XII 1960)

The problem of a strong explosion in an ideal gas was solved by L. I. Sedov ([1]). The non-self-similar problem of a point explosion in an ideal gas was considered by a number of authors ([2–5]). The problem of a strong point explosion in an ideal compressible medium was considered by N. N. Kochina and N. S. Mel' nikova ([6]), and also by Yu. L. Yakimov ([10]); the non-self-similar problem of a point explosion—by N. N. Kochina ([7,8]). It is of interest to study the dependence of the solution of the explosion problem in an ideal two-parameter medium on the explosion energy E_0, on the initial pressure p_1, and on the initial density ρ_1; here it is important to clarify the various cases in which the solution found for definite values of the parameters E_0, p_1, ρ_1 can be recalculated for other values of the explosion energy E_0 and other initial parameters p_1 and ρ_1*. The present article is devoted to this question and to the asymptotic behavior of the solutions.

The internal energy as a function of the pressure and density of any ideal two-parameter medium can always be represented in the form

$$\varepsilon(p,\rho) = \frac{p}{\rho}\,\varphi\left(\frac{p}{p_1}, \frac{\rho}{\rho_1}, \frac{p_1^*}{p_1}, \frac{p_2^*}{p_1}, \dots, \frac{p_n^*}{p_1}, \frac{\rho_1^*}{\rho_1}, \frac{\rho_2^*}{\rho_1}, \dots, \frac{\rho_m^*}{\rho_1}\right), \tag{1}$$

where φ is a dimensionless function of its arguments; p_1, ρ_1 are the initial pressure and density of the medium; p_i^* and ρ_i^* are certain physical constants with the dimensions of pressure and density. Representation (1) is valid for many complex reversible processes, and, in particular, when dissociation, ionization, or phase transitions occur.

The equations of one-dimensional adiabatic unsteady motion of an ideal medium have the form

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0, \qquad \frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial r} + \frac{(\nu-1)\rho v}{r} = 0,$$

$$\left(\frac{\partial\varepsilon}{\partial\rho}-\frac{p}{\rho^2}\right)\frac{d\rho}{dt}+\frac{\partial\varepsilon}{\partial p}\frac{dp}{dt}=0, \tag{2}$$

where v is the velocity; t is the time; r is the Eulerian coordinate; $\nu = 1$ for plane waves, $\nu = 2$ for cylindrical waves, and $\nu = 3$ for spherical waves. Dimensional constants can enter into (2) only through (1), and also through the initial and boundary conditions.

The conditions on the shock wave propagating through the undisturbed medium ($v_1 = 0$) are written as follows:

$$-\rho_1 c = \rho_2(v_2 - c), \qquad \rho_1 c^2 + p_1 = \rho_2(v_2 - c)^2 + p_2,$$

$$\varepsilon_2 - \varepsilon_1 = \frac{1}{2}(p_1 + p_2)(1/\rho_1 - 1/\rho_2) \tag{3}$$

(the quantities at the shock front are denoted by the subscript 2).

* The conclusions given below have not been formulated or used in the general case for compressible media. The ambiguities existing in these questions have often led to erroneous assertions.

If the solution is continued to the center of the explosion, then the condition at the center has the form*

$$v(0,t) = 0. \tag{4}$$

The system of determining parameters for the problem of a point explosion in an ideal two-parameter medium is represented by the quantities

$$r,\ t,\ E_0,\ p_1,\ \rho_1,\ \nu,\ p_1^*,\ p_2^*,\dots,\ p_n^*,\ \rho_1^*,\ \rho_2^*,\dots,\ \rho_m^*, \tag{5}$$

where E_0 is the energy released at the center of symmetry at the instant of explosion. From these parameters one can form the following dimensionless combinations:

$$l=\frac{r}{r_0},\quad \tau=\frac{t}{t_0},\quad \nu,\quad \frac{p_1^*}{p_1},\quad \frac{p_2^*}{p_1},\dots,\frac{p_n^*}{p_1},\quad \frac{\rho_1^*}{\rho_1},\quad \frac{\rho_2^*}{\rho_1},\dots,\frac{\rho_m^*}{\rho_1}; \tag{6}$$

$$r_0 = (E_0/p_1)^{1/\nu}, \qquad t_0 = E_0^{1/\nu}\rho_1^{1/2}p_1^{-(\nu+2)/2\nu}. \tag{7}$$

Instead of the dimensionless variable parameters l and τ indicated in table (6), one may take the equivalent dimensionless variable parameters λ and q in the form

$$\lambda = r/r_2, \qquad q = a_1^2/c^2, \tag{8}$$

where a_1 is the speed of sound in the undisturbed medium, r_2 is the radius of the shock wave, and c is the speed of the shock wave.

It should be noted that if the internal energy is a linear function of the pressure,

$$\varepsilon(p,\rho) = \frac{p_1^*}{\rho_1^*}\left[P\varphi(R) + \Delta(R)\right]\left(R = \frac{\rho}{\rho_1^*},\ P = \frac{p}{p_1^*}\right), \tag{9}$$

then from conditions (3) we obtain the dependences of P_2, f_2, and q on R_2 in explicit form:

$$P_2 = P_1 + \frac{\Delta(R_1) - \Delta(R_2) + P_1[\varphi(R_1) - \varphi(R_2) + 1/R_2 - 1/R_1]}{\varphi(R_2) - \frac{1}{2}(1/R_1 - 1/R_2)}, \quad f_2 = 1 - \frac{R_1}{R_2}, \tag{10}$$

$$q = \frac{B(1 - R_1/R_2)\{R_1[\varphi(R_2) + 1/2R_2] - 1/2\}}{P_1[\varphi(R_1) - \varphi(R_2)] + \Delta(R_1) - \Delta(R_2) + P_1(1/R_1 - 1/R_2)}\left(B = \frac{\rho_1^* a_1^2}{p_1^*},\ f = \frac{v}{c}\right).$$

All dimensionless functions determining the motion of the perturbed medium can depend only on two independent dimensionless variables l and τ (or λ and q) and on the abstract constant parameters

$$\nu,\ \frac{p_1^*}{p_1},\ \frac{p_2^*}{p_1}, \dots, \frac{p_n^*}{p_1},\ \frac{\rho_1^*}{\rho_1},\ \frac{\rho_2^*}{\rho_1}, \dots, \frac{\rho_m^*}{\rho_1}.$$

From consideration of the table of dimensionless parameters (6)–(8), on which the dimensionless solution of the problem depends, it is clear that the solution depends on the energy of the explosion only through the variables l and τ (or λ and q). Consequently, having a solution of the problem for a particular initial explosion energy $E_0^{(1)}$, one can recalculate it for any explosion energy $E_0^{(2)}$; moreover, in order that the dimensionless variables remain the same, it is necessary to recalculate r and t, as well as all quantities having the dimensions of length and time, according to the formulas

$$r^{(2)} = (E_0^{(2)}/E_0^{(1)})^{1/\nu} r^{(1)}, \qquad t^{(2)} = (E_0^{(2)}/E_0^{(1)})^{1/\nu} t^{(1)}, \tag{11}$$

where $r^{(1)}$ and $t^{(1)}$ are the coordinates corresponding to the value of the explosion energy $E_0^{(1)}$, and $r^{(2)}$ and $t^{(2)}$ to the value $E_0^{(2)}$. Under such a recalculation from one energy $E_0^{(1)}$ to another $E_0^{(2)}$, there is no need to recalculate anew the dimensionless characteristics of the motion; it is sufficient to compute them only once. From the form of the internal energy (1) and the dimensionless parameters (6) it follows that the parameters p_1 and ρ_1 enter not only into the expressions for l and τ (or λ and q), but also into other parameters, on which only the function φ depends. These parameters do not enter into the expression for the function φ only in the case when the function φ reduces to a constant, i.e., the considered

* If the solution is obtained with a cavity, then an analogous condition for the pressure holds at the boundary

the medium is an ideal gas. In this case the solution of the problem obtained for specific values of the initial parameters $E_0^{(1)}, p_1^{(1)}, \rho_1^{(1)}$ can be recalculated for any initial parameters $E_0^{(2)}, p_1^{(2)}, \rho_1^{(2)}$ by the corresponding recalculation of r and t according to the formulas:

$$r^{(2)} = \left(E_0^{(2)} \rho_1^{(1)} / E_0^{(1)} \rho_1^{(2)}\right)^{1/\nu} r^{(1)},$$

$$t^{(2)} = \left(E_0^{(2)} / E_0^{(1)}\right)^{1/\nu} \left(p_1^{(2)} / p_1^{(1)}\right)^{-(\nu+2)/2\nu} \left(\rho_1^{(2)} / \rho_1^{(1)}\right)^{1/2} t^{(1)}, \tag{12}$$

where $r^{(1)}$ and $t^{(1)}$ correspond to the values of the initial parameters $E_0^{(1)}, p_1^{(1)}$ and $\rho_1^{(1)}$, while $r^{(2)}$ and $t^{(2)}$ correspond to the values $E_0^{(2)}, p_1^{(2)}$ and $\rho_1^{(2)}$ (12).

If the internal energy of the medium can be represented in the form

$$\varepsilon = \frac{p}{\rho} \varphi \left(\frac{\rho}{\rho_1}, \frac{\rho_1^*}{\rho_1}, \frac{\rho_2^*}{\rho_1}, \dots, \frac{\rho_m^*}{\rho_1}\right),$$

then the solution of the problem obtained for specific values of the parameters $E_0^{(1)}, p_1^{(1)}$ can be recalculated for any $E_0^{(2)}, p_1^{(2)}$, assuming that the parameter ρ_1 does not change. In this case the variables r and t are recalculated according to formulas (12), in which one should set $\rho_1^{(1)} = \rho_1^{(2)}$. Similarly, if the internal energy of the medium can be represented in the form

$$\varepsilon = \frac{p}{\rho} \varphi \left(\frac{p}{p_1}, \frac{p_1^*}{p_1}, \frac{p_2^*}{p_1}, \dots, \frac{p_n^*}{p_1}\right),$$

then the solution of the problem for specific values of the parameters $E_0^{(1)}, \rho_1^{(1)}$ can be recalculated for any values $E_0^{(2)}, \rho_1^{(2)}$, with r and t correspondingly recalculated according to formulas (12), in which one should set $p_1^{(1)} = p_1^{(2)}$. No such considerations can be made regarding the dimensionless parameter ν; consequently, for each of the three values of ν the corresponding calculation must always be carried out.

If a non-point explosion is considered, then to the determining parameters (5) one should add the parameter r_0^*—the initial radius of the explosion products, with the dimension of length—and the constants p_i^*, ρ_i^* $(n+1 \le i \le k,\ m+1 \le i \le s)$, which characterize the properties of the explosive substance. The table of dimensionless determining parameters (6) in this case takes the form

$$l = \frac{r}{r_0}, \quad \tau = \frac{t}{t_0}, \quad \frac{r_0^*}{r_0}, \quad \nu, \quad \frac{p_1^*}{p_1}, \quad \frac{p_2^*}{p_1}, \dots, \frac{p_k^*}{p_1}, \quad \frac{\rho_1^*}{\rho_1}, \quad \frac{\rho_2^*}{\rho_1}, \dots, \frac{\rho_s^*}{\rho_1}, \tag{13}$$

where r_0 and t_0 are determined by formulas (7). From table (13) it is evident that, in the general case, recalculation to other values of the explosion energy E_0 is impossible, since the energy E_0 enters not only into the expression for the parameters l and τ (or λ and q), but also into the expression for the parameter r_0^*/r_0. Recalculation is possible only in the case when, simultaneously with the change in the explosion energy, the initial parameter r_0^* changes, and in such a way that the parameter r_0^*/r_0 remains unchanged, i.e., the formula

$$r_0^{*(2)} = \left(E_0^{(2)}/E_0^{(1)}\right)^{1/\nu} r_0^{*(1)} \tag{14}$$

holds. In this case all the remaining parameters entering table (13) must remain unchanged. Thus, the initial density and pressure must not change; only the quantity r_0^*, which determines the size of the explosive substance, changes.

In Brode' s article (11) it is asserted that, for the problem considered by the author of the explosion of a spherical charge in real air with allowance for dissociation and for the initial radius of the explosive substance, recalculation is possible from one set of values of the explosion energy, initial pressure, and dimensions of the explosive substance to other values of these quantities. It turns out that recalculation to new values of the explosion energy E_0 is possible only in the case when the radius of the explosive substance changes according to law (14). Recalculation to new values of the initial pressure p_1 is, for the internal energy used in the article, altogether impossible.

To obtain a complete picture of the motion, one must carry out the calculation for large times. For this one can use asymptotic formulas. If it follows from the calculation of a particular problem on a point explosion that, for large times t, the shock-wave velocity c is close to the speed of sound in the undisturbed medium a_1, then one can use the acoustic asymptotic formula $r_2 = a_1 t$.

The pressure at the shock front depends on the shock-wave radius in the following way: $p_2/p_1 = f(r_2/r_0)$; $p_2/p_1 = f(a_1(\rho_1/p_1)^{1/2} t/t_0)$. Analogous formulas can be written for the dimensionless quantities of velocity and density.

Let us derive the conditions imposed on media for which, in a point explosion, beginning from some instant of time, the motion in the entire disturbed region, with the exception of some neighborhood of the shock wave, can be approximated by the motion of an incompressible fluid ($\rho = \rho_0$). From the third equation of system (2) it is clear that in the case of an incompressible fluid we have the equality

$$\partial\varepsilon(p, \rho_0)/\partial p = 0. \tag{15}$$

Let us note that if the function $\varepsilon(p, \rho)$ in the neighborhood of the value $\rho = \rho_0$ has the form

$$\varepsilon(p, \rho) = -p/\rho + f(p), \tag{16}$$

then equality (15) does not in general entail the equality $\rho = \rho_0$.

If in the medium under consideration equation (15) may be regarded as satisfied (while equation (16) is not satisfied), then in the case when the solution is obtained with a cavity, the solution of the problem on a point explosion in an incompressible fluid (1) may be taken as the asymptotic form; if, however, the solution reaches the center of symmetry, the asymptotic form may be taken as

$$v = 0, \qquad \rho = \rho_0, \qquad p = \psi(t). \tag{17}$$

The form of the function $\psi(t)$ is determined from the solution of the problem.

In paper ([6]), as an example, a strong point explosion is considered in a medium with internal energy of the form $\varepsilon(p, \rho) = p(\rho - \rho_0)/2\rho^2$. From the graphs presented there of the dimensionless characteristics v/v_2, ρ/ρ_2, and p/p_2 as functions of λ, it is seen that compressibility is significant only near the shock wave; near the cavity the fluid behaves as incompressible. At the same time, for a number of values of the initial parameter $R_1 = \rho_1/\rho_0$, the asymptotic formulas have the form (17), and for a number of them they have the form corresponding to the solution of the self-similar problem on a point explosion in an incompressible fluid (1).

If the motion is self-similar, i.e., the internal energy can be represented in the form $\varepsilon(p, \rho) = p\varphi(\rho/\rho_0)/\rho_0$, then condition (16) is written as follows:

$$\varphi(\rho/\rho_0) = (\rho - \rho_0)/\rho. \tag{18}$$

In paper ([6]) it is noted that, for a function $\varphi(\rho/\rho_0)$ of the form (18), the solution of the problem cannot be approximated by the solution for an incompressible fluid.

Steklov Mathematical Institute
Academy of Sciences of the USSR

Received
3 XII 1960

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Note: Figure translations are in progress. See original paper for figures.