PHYSICS

V. D. KUKIN and A. R. FRENKIN

1961

SovietRxiv

Abstract

Full Text

PHYSICS

V. D. KUKIN and A. R. FRENKIN

ON THE CONSTRUCTION OF THE SCATTERING MATRIX IN NONLOCAL THEORIES

(Presented by Academician N. N. Bogolyubov, 8 IV 1961)

1. Questions of the construction of the S-matrix in theories of nonlocal type have already been discussed many times. We shall consider some possibilities for introducing changes of a "nonlocal" character into the S-matrix scheme existing in local theory ([1]). In doing so we wish to preserve the graph technique, and therefore shall start from the representation of the local S-matrix in the form

$$S = T\left(\exp\left\{i\int \mathcal{L}(x)\,dx\right\}\right). \tag{1}$$

2. In a theory with an indefinite metric ([2]) there exists a unitary "complete" S-matrix, connecting both "physical" and "nonphysical" states, and it is necessary to construct the "physical" scattering matrix, connecting only "physical" states. The matrix $S' = PSP$ (where P is the projection operator onto "physical" states), obtained by the simple projection of S onto the subspace of "physical" states, turns out to be nonunitary,

$$S'^{+}S' = PS^{+}PSP \neq P. \tag{2}$$

Indeed, in the expansion of (2) by Wick' s theorem only those terms contribute in which all operators of "nonphysical" fields are contracted; moreover S^{+} and S, separated by the operator P, can be connected only by "physical" contractions (contractions of operators of "physical" fields), while inside S^{+} and S one must take "complete" contractions, including both the "physical" and the "nonphysical" parts. Since inside S^{+} and S one must take chronological contractions (causal functions), and between S^{+} and S—ordinary contractions (negative-frequency parts of commutator functions), then in the expansion of (2) by Wick' s theorem we have different rules for writing chronological and ordinary contractions. Therefore the nonunitarity of S' can be interpreted as a consequence of a disagreement between chronological and ordinary contractions.

3. In constructing the S-matrix in a theory with nonlocal interaction ([3]), a rupture of the usual connection between the definition of the T-product and the definition of the ordinary product of field operators naturally occurs, since the notion of a chronologically ordered sequence has no meaning for operators depending on an entangled group of arguments. The supplementary definition of the T-product by Wick' s theorem as the sum of all possible normal products with all possible chronological contractions, adopted in ([3]), in essence ignores this rupture, which is immediately manifested in the nonunitarity of the S-matrix constructed in this way.

4. The examples considered suggest an attempt to obtain a scattering matrix of "nonlocal" type on the basis of the disagreement between the definition of chronological contraction and the definition of ordinary contraction. In what way would it be most convenient to carry out this idea?

Any Feynman graph with fixed external lines depends only on the vertices and internal lines. From this point of view, changes in the value of a graph in a theory with nonlocal interaction can be explained if the introduction of a form factor into the interaction Lagrangian is interpreted as a change of the graph vertices. There is also another possibility for changing the value of a graph, namely: without changing the graph vertices, i.e., using a local Lagrangian, to change the internal lines. For this it is sufficient to change only the definition of chronological contractions. On the other hand, from the standpoint of a theory with an indefinite metric, it is more advantageous not to change the definitions of the usual contraction, since it is precisely the appearance of "wrong" signs in the commutation function that creates difficulties in the physical interpretation of the corresponding quanta.

Therefore we implement the idea of inconsistency in the following way. A new, "nonlocal" definition of chronological contraction is postulated without changing the definition of the usual contraction, i.e., the "nonlocal" causal function $\widetilde{D}^c(x)$ is no longer related to $D^{(-)}(x)$ in the usual, "local" way:

$$\widetilde{D}^c(x) \neq \theta(x^0)D^{(--)}(x) + \theta(-x^0)D^{(-)}(-x) = D^c(x). \tag{3}$$

The "nonlocal" matrix S' is represented in the form

$$S' = \widetilde{T}\left(\exp\left\{i\int \mathcal{L}(x)\,dx\right\}\right), \tag{4}$$

where the operation of "nonlocal" T-product is defined by means of Wick expansion; the $\widetilde{T}$-product is represented as the sum of all possible normal products with all possible "nonlocal" chronological contractions. For ordinary products the usual Wick theorem remains valid.

The inconsistency of the definitions of chronological and ordinary contractions immediately leads to the nonunitarity of the "nonlocal" matrix S'.

5. It should be noted that inconsistency of the causal and commutation functions occurs in quantum electrodynamics, where, by means of a gauge transformation of the second kind, one can obtain an expression for the chronological contraction of electromagnetic potentials containing an arbitrary gauge of the longitudinal part [(1)] and not consistent with the expression for the commutation function, which is not changed under such a transformation. However, in this case, as a consequence of the condition of gauge invariance, the S-matrix not only remains unitary, but in general is not changed.

6. For all the examples considered, the difficulty associated with violation of the unitarity of the scattering matrix is common, and in the theory one must additionally indicate a method that makes it possible to restore the lost unitarity. The exception is the case of quantum electrodynamics, where the additional condition of gauge invariance guarantees not only unitarity, but even the invariance of the S-matrix. In the theory with an indefinite metric one can also formulate an additional condition [(2)] that makes it possible to construct a unitary "physical" matrix $\widetilde{S}$.

We shall apply the method of unitarization of the scattering matrix proposed in [(3)] and suitable in all the cases considered above.

Let us explain its idea by an example. In the case of a quantized electromagnetic field interacting with a given distribution of classical currents $J_\nu(x)$ [(4)], the S-matrix will be nonunitary. Indeed, for processes without the participation of photons,

$$S = \exp\left\{-\frac{1}{2}\iint dx\, dy\, J_\nu(x) D^c(x-y) J_\nu(y)\right\} \tag{5}$$

is not a pure phase, since D^c contains both an imaginary and a real part. However, this nonunitarity is easily removed by introducing into the exponent

exponents, the corresponding real addition R, which in this case will simply be a c-number.

In the general case the addition R must be a Hermitian operator. In our case, as also in the case of the theory with nonlocal interaction ([3]), one can in this way unitarize the nonunitary "nonlocal" matrix S' (4) and formally write the expression for the unitary "nonlocal" scattering matrix in the form

$$\widetilde{S} = \widetilde{T}\left(\exp\left\{i\int \mathcal{L}(x)\, dx + \int R(x)\, dx\right\}\right), \tag{6}$$

where $R(x)$ is the corresponding Hermitian operator. In reality it is not possible to obtain a closed expression for R, in view of its complicated operator nature. However, to each operator function $S_n(x_1 \ldots x_n)$ of the functional expansion ([1]) of the matrix S' in powers of the coupling constant one may add a Hermitian operator $R_n(x_1 \dots x_n)$ such as to ensure the fulfillment of the unitarity condition in that order. Thus the operator R is determined in the form of a chain of successively found operator terms R_n. It is important to note that each term R_n of this chain is determined uniquely from the unitarity condition in the given order through the operator functions $\widetilde{S}_n$ of lower indices.

If the unitarization operation is not needed, as, for example, in the case of quantum electrodynamics, then the operator R will be equal to zero.

In addition to the chain of unitarizing Hermitian operators R_n, we are entitled to specify, as in the case of a local theory ([1]), a chain of quasilocal anti-Hermitian operators $i\Lambda_n(x_1 \dots x_n)$; moreover, owing to the "nonlocal" character of the causal functions, all counterterms will be finite.

7. We shall illustrate the proposed scheme for obtaining the "nonlocal" scattering matrix $\widetilde{S}$ by the example of a neutral pseudoscalar meson theory with the interaction Lagrangian

$$\mathcal{L}(x) = g\bar{\psi}(x)\gamma_5\psi(x)\varphi(x). \tag{7}$$

First of all one must specify the form of the "nonlocal" chronological pairing. For simplicity we choose the "nonlocal" causal functions $\widetilde{D}^c$ and $\widetilde{S}^c$, regularized according to Pauli–Villars,

$$\widetilde{D}^c(\mu, \nu; x) = D^c(\mu; x) - D^c(\nu; x); \tag{8}$$

$$\widetilde{S}^c(m, M; x) = (i\hat{\partial} + m)\{D^c(m; x) - D^c(M; x)\}. \tag{9}$$

To accuracy up to the second order of perturbation theory,

$$\widetilde{S}_1(x) = i\mathcal{L}(x); \qquad R_1(x) \equiv 0; \tag{10}$$

$$\widetilde{S}_2(x, y) = i^2\widetilde{T}(\mathcal{L}(x)\mathcal{L}(y)) + R_2(x, y) + i\Lambda_2(x, y), \tag{11}$$

where, from the unitarity condition in the second order,

$$2R_2(x, y) = \widetilde{T}(\mathcal{L}(x)\mathcal{L}(y)) + \widetilde{T}^{+}(\mathcal{L}(x)\mathcal{L}(y)) - \mathcal{L}(x)\mathcal{L}(y) - \mathcal{L}(y)\mathcal{L}(x). \tag{12}$$

From this it is easy to calculate the "nonlocal" corrections of the second order, for example the "nonlocal" correction $\widetilde{\Pi}(k^2)$ to the polarization operator, which appears as a consequence of the "nonlocal" character of the nucleon causal functions in the nucleon loop.

8. In conclusion let us dwell on questions of causality. The proposed scheme of a "nonlocal" scattering matrix satisfies the causality condition in a weakened form, which may be formulated as follows. Let the complete system consist of two sufficiently far separated ⋯

a large distance between the parts, and the interaction between the parts is negligibly small. Then one may assume that the field operators from one part of the system commute (anticommute) with the field operators from the other part of the system. In this case the Lagrangian of the complete system splits into a sum of mutually commuting Lagrangians of the parts of the system: $\mathcal{L} = \mathcal{L}_1 + \mathcal{L}_2$, and the matrix S' for the complete system will be the product of the matrices S_i for the parts of the system

$$S' = \widetilde{T}\left(\exp\left\{i\int \mathcal{L}\,dx\right\}\right) = \widetilde{T}\left(\exp\left\{i\int \mathcal{L}_1\,dx\right\}\right)\widetilde{T}\left(\exp\left\{i\int \mathcal{L}_2\,dx\right\}\right) = S_1'S_2'. \tag{13}$$

On passing to the unitary matrices $\widetilde{S}$, the unitarizing operator R likewise splits into the sum $R = R_1 + R_2$, where both R_1 and R_2 are determined from the unitarity condition independently of one another, only through the field operators of their own part of the system. Then, obviously, $\widetilde{S} = \widetilde{S}_1\widetilde{S}_2$.

The question of satisfying stronger causality conditions within the framework of the proposed scheme requires additional study.

The authors express their deep gratitude to N. N. Bogolyubov for his constant attention and valuable suggestions, and to B. V. Medvedev, M. K. Polivanov, D. A. Slavnov, and A. D. Sukhanov for useful discussions.

Moscow State University
named after M. V. Lomonosov

Received
7 IV 1961

REFERENCES

[1] N. N. Bogolyubov, D. V. Shirkov, *Introduction to the Theory of Quantized Fields*, Moscow, 1957.
[2] N. N. Bogolyubov, B. V. Medvedev, M. K. Polivanov, *Scientific Reports of the Higher School, Physico-Mathematical Sciences*, 2, 137 (1958).
[3] B. V. Medvedev, *JETP*, **32**, 87 (1957).
[4] R. Glauber, *Phys. Rev.*, **84**, 395 (1951).

Note: Figure translations are in progress. See original paper for figures.