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MECHANICS

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Abstract

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LINEARLY ENVELOPING COUPLER CURVES OF A FOUR-LINK ELLIPSOGRAPH

Figure 1 shows the most general case of a four-link ellipsograph, whose sliders 1 and 3 slide along fixed guides forming an arbitrary constant angle α. Let us consider the question of the equations of the linearly enveloped coupler curves of coupler 2. For this purpose, in the plane of coupler 2 choose an arbitrary straight line $u - u$. Next find the instantaneous center of rotation P of coupler 2 and drop from point P the perpendicular PM to the straight line $u-u$. Point M will belong to the curve linearly enveloping the straight line $u - u$. From point A drop the perpendiculars AE and AF to the directions BC and $u - u$, and introduce the notation $AF = p$, $AE = p_1$, and $BC = a$. From the general equation for the family of straight lines $u = u$

$$x \cos\theta + y \sin\theta = p \tag{1}$$

or

$$x \cos(\theta_1 + \gamma) + y \sin(\theta_1 + \gamma) = p_1 \cos\gamma - B \sin\gamma, \tag{2}$$

where

$$B = \frac{1}{2}\left[a - \sqrt{a^2 - 4p_1 \operatorname{ctg}\alpha + 4p_1^2}\right].$$

The partial derivative with respect to the parameter θ_1 of expression (2) will have the form

$$y \cos(\theta_1 + \gamma) - x \sin(\theta_1 + \gamma) = \frac{\partial p_1}{\partial \theta_1} \cos\gamma - A \frac{\partial p_1}{\partial \theta_1} \sin\gamma, \tag{3}$$

where

$$A = \frac{a \operatorname{ctg}\alpha + 2p_1}{\sqrt{a^2 - 4p_1 \operatorname{ctg}\alpha + 4p_1^2}}.$$

The segment p_1 will be equal to

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$$p_1 = \frac{a}{2\sin\alpha}\left[\cos\alpha + \cos(2\theta_1 - \alpha)\right]. \tag{4}$$

The partial derivative $\partial p_1/\partial\theta_1 = \Psi$ will be equal to

$$\Psi = -\frac{a}{\sin\alpha}\sin(2\theta_1 - \alpha). \tag{5}$$

From equations (2) and (3) we obtain the parametric equations of the desired linearly enveloping coupler curve

$$\begin{aligned} x = {} & [p_1\cos(\theta_1+\gamma) - \Psi\sin(\theta_1+\gamma)]\cos\gamma + \\ & + [B\cos(\theta_1+\gamma) - A\Psi\sin(\theta_1+\gamma)]\sin\gamma, \end{aligned} \tag{6}$$

$$\begin{aligned} y = {} & [p_1\sin(\theta_1+\gamma) + \Psi\cos(\theta_1+\gamma)]\cos\gamma + \\ & + [B\sin(\theta_1+\gamma) + A\Psi\cos(\theta_1+\gamma)]\sin\gamma, \end{aligned}$$

where p_1 and Ψ are determined by equations (4) and (5).

If the angle $\gamma = 0$, then equations (6) take the form

$$\begin{aligned} x &= p_1\cos\theta_1 - \Psi\sin\theta_1, \\ y &= p_1\sin\theta_1 + \Psi\cos\theta_1. \end{aligned} \tag{7}$$

The curve described by equations (7) will be a curve of astroid type. The axes of symmetry of this curve make angles $\alpha/2$ and $90^\circ + \alpha/2$ with the axes Ax and Ay. The curve itself is, as it were, a deformed astroid of general form, compressed in the direction of the axis making the angle $90^\circ + \alpha/2$, and stretched in the direction of the axis making the angle $\alpha/2$. If $\alpha = 90^\circ$, then we obtain the mechanism of an ellipsograph with link BC, whose points B and C move along two mutually perpendicular straight lines.

Let us show that the general case of the ellipsograph mechanism considered by us, shown in Fig. 1, can be reduced to the consideration of an equivalent ellipsograph mechanism whose ends of the connecting rod move along two mutually perpendicular straight lines. To do this, join point P to point A and construct on the segment PA a circle q of radius OA. It follows from the construction that this circle will pass through the points C and B. The radius OA of this circle will be equal to $OA = a/2\sin\alpha$. Join point C to point O and find, at the intersection of the direction CO with the circle q, the point K. Join point K to point A. It is not difficult to see that the mechanism considered above by us, consisting of the moving links $1, 2$, and 3, can be replaced by an equivalent mechanism consisting of the moving links $1, 2'$, and $3'$, while the sliders 1 and $3'$ move along the mutually perpendicular axes Ax' and Ay'.

Fig. 1

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Fig. 1

Figure 1: Fig. 1

The second equivalent mechanism is a crank-and-connecting-rod mechanism with crank $1'$, connecting rod $2'$, and slider 1, in which the lengths AO and AC of the crank $1'$ and the connecting rod $2'$ are equal. The equations for the linearly enveloped connecting-rod curves of a mechanism of this kind were considered by us earlier ([1]).

If point A is taken as the pole of the podaire, then the locus of the points F forms the podaire of the curve linearly enveloping the straight line $u-u$, and the locus of the points E forms the podaire of the curve linearly enveloping the straight line CB. The equations of these podaires in polar form will be $p = p(\theta_1)$ and $p_1 = p_1(\theta_1)$, where the function $p_1(\theta_1)$ is determined by equation (4), and the function $p = p(\theta_1)$ is equal to

$$p = p_1 \cos\gamma - \frac{1}{2}\left[a - \sqrt{a^2 - 4p_1 \operatorname{ctg}\alpha + 4p_1^2}\right]\sin\gamma.$$

In the general case the podaires under consideration will be curves of rose type, but deformed in two mutually perpendicular directions along the axes Ax' and Ay'.

We now pass to the consideration of the mechanism of the inverse ellipsograph (Fig. 2). Let us find the instantaneous center of rotation P of the angular connecting rod 2 and drop onto an arbitrarily chosen straight line $u-u$ in the plane of the connecting rod 2 the perpendicular PM. The point M will belong to the curve linearly enveloping the straight line $u-u$. From point A we drop onto the direction $u-u$ and CG the perpendiculars AF and AE. Introduce the notation: $AF = p$, $AE = p_1$, $AB = a$, $AD = b$, and $BC = c$, where AD and BC are the shortest distances of the sides of the rigid angle α from the points A and B.

The equation of the family of straight lines $u-u$ will be

$$x\cos\theta + y\sin\theta = p \tag{8}$$

or

$$x\cos(\theta_1+\gamma) + y\sin(\theta_1+\gamma) = b\frac{\sin\gamma}{\sin\alpha} + p_1\frac{\sin(\alpha+\gamma)}{\sin\alpha}. \tag{9}$$

The partial derivative with respect to the parameter θ_1 of equation (9) is equal to

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Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

$$y\cos(\theta_1+\gamma) - x\sin(\theta_1+\gamma) = \frac{\sin(\alpha+\gamma)}{\sin\alpha}\frac{\partial p_1}{\partial\theta_1}. \tag{10}$$

The segment p_1 is equal to

$$p_1 = a\cos\theta_1 + c. \tag{11}$$

Fig. 2

Fig. 3

Consequently,

$$\frac{\partial p_1}{\partial\theta_1} = -a\sin\theta_1. \tag{12}$$

From equations (9) and (10), taking into account conditions (11) and (12), we obtain the parametric equations of the required linearly enveloped coupler curve

$$\begin{aligned} x &= A\cos\gamma + B\cos(\theta_1+\gamma), \\ y &= A\sin\gamma + B\sin(\theta_1+\gamma), \end{aligned} \tag{13}$$

where the constants A and B are equal to:

$$A = a\frac{\sin(\alpha+\gamma)}{\sin\alpha}, \qquad B = b\frac{\sin\gamma}{\sin\alpha} + c\frac{\sin(\alpha+\gamma)}{\sin\alpha}.$$

Passing to a rectangular coordinate system, we obtain the equation of the linearly enveloped curve in the form

$$x^2 + y^2 - 2A\cos\gamma\, x - 2A\sin\gamma\, y + (A^2 - B^2) = 0. \tag{14}$$

Thus, the linearly enveloped coupler curve will be a circle.

If the dimensions of the mechanism satisfy the condition $b = c = 0$, then we obtain the mechanism shown in Fig. 3. For this mechanism we obtain the following parametric equations of the linearly enveloped curve:

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$$x = A\cos\gamma, \qquad y = A\sin\gamma, \tag{15}$$

i.e., this curve degenerates into a point. Consequently, the straight line u—u (Fig. 3) will always pass through the point M, belonging to the circle q, which passes through the points P, C, A, and B. The equation of the circle q will be

$$x^2 + y^2 - ax - a\cot\alpha\, y = 0. \tag{16}$$

The radius r of the circle q is equal to $r = a/2\sin\alpha$.

From the indicated property follows the possibility of obtaining an equivalent mechanism (Fig. 3), consisting of links $1, 2$, and $3'$, having the angle $\angle ACB' = 90^\circ$ and the distance $AB' = a/\sin\alpha$. The polodes described by the points F and E will be circles.

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CITED LITERATURE

1. I. I. Artobolevsky, DAN, **139**, No. 5 (1961).

Note: Figure translations are in progress. See original paper for figures.

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