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Abstract

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GEOPHYSICS

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ON THE RELATION BETWEEN THE FLUCTUATION SPECTRUM OF A RADAR SIGNAL AND THE MOTION OF SCATTERERS IN METEOROLOGICAL OBJECTS

(Presented by Academician E. K. Fedorov on 31 III 1961)

Questions of the scattering of radio waves by an ensemble of moving scatterers have been considered by a number of authors. However, the results obtained cannot be applied to solving the question of the relation between the fluctuation spectrum after detection of a radar signal and the velocity distribution of scattering particles that are nonuniform in size. The works carried out at the Institute of Radio Engineering and Electronics of the Academy of Sciences of the USSR under the direction of G. S. Gorelik are closest to the question considered here ([1–3]).

In the present article a method of calculation is set forth that is also suitable for the case of scattering of radio waves by an ensemble of particles having some distribution in size. The instantaneous value of the field received from an ensemble of such scatterers may be written in the form

$$E(t) = \sum_{i=1}^{N} a_i \cos[\omega t - \alpha_i(t)] = \sum_{i=1}^{N} a_i \cos[\omega t - 2k\xi_i(t)],$$

where a_i is the amplitude of the signal from the i-th scatterer; ω is the frequency emitted by the radar station; ξ_i is the projection of the radius vector of the i-th particle onto the direction of the beam.

The power, averaged over the period of the high frequency, characterizes the signal at the output of the square-law detector.

$$I(t) = \overline{E^2(t)} = \sum_{i=1}^{N}\sum_{j=1}^{N} a_i a_j \cos[\alpha_i(t) - \alpha_j(t)].$$

Suppose that in the interval from 0 to π the phases α are distributed uniformly. Performing averages analogous to those carried out in ([1]), we obtain an expression for the correlation function of the intensity of the scattered field z:

$$\overline{z(t)z(t)} = \sum_{i=1}^{N}\sum_{j=1}^{N}\overline{a_i^2 a_j^2 \cos\Delta_s\varphi_{ij}}, \tag{1}$$

where

$$\Delta_s\varphi_{ij} = 2k\{[\xi_i(t+s)-\xi_i(t)] - [\xi_j(t+s)-\xi_j]\}.$$

For the case of turbulent diffusion it has been shown ([3]) that the correlation function $\overline{z(t)z(t+s)}$ falls practically to zero in such a time s during which the velocities of the scatterers may be regarded as constant. In gravitational fall of the scatterers this condition is all the more fulfilled. Therefore, in the expression for the correlation function one may pass to the particle velocities:

$$\xi_i(t+s) - \xi_i(t) = v_i(t)s,$$

where v_i is the projection of the velocity of the i-th particle onto the direction of the beam. If pairs of scatterers are statistically equivalent, then the summation sign may be omitted, and in this case the correlation function takes the form

$$\overline{z(t)z(t+s)} = N^2\overline{a_i^2 a_j^2 \cos 2k(v_i - v_j)s'}. \tag{2}$$

The projection of the particle displacement velocity onto the beam direction may be represented as the sum of the projection of the fall velocity and the velocity of transport of the particle by the air flow: $v_i = v_{ir} + v_{it}$, where v_{ir} is the projection of the fall velocity; v_{it} is the projection of the transport velocity of the i-th particle in the beam direction. The magnitude of the signal reflected by the i-th scatterer is expressed through its fall velocity: $a_i^2 = C[\varphi(v_{ir})]^2$, since both these quantities are functions of the particle size: $a_i^2 = cr_i^6$, $r_i = \varphi(v_{ir})$. The gravitational fall velocities of the particles and the velocities of transport by the flow are independent. Expanding the expression for the correlation function, we obtain:

$$\begin{aligned}\overline{z(t)z(t+s)} &= N^2\overline{a_i^2 a_j^2}\cos 2k(v_i - v_j)s \\ &= N^2C^2\int_0^\infty\int_0^\infty\int_{-\infty}^{+\infty}[\varphi(v_{ir})]^6[\varphi(v_{jr})]^6\cos 2k(v_{ir} - v_{jr} + v_{it} - v_{jt}) \\ &\quad\times w(v_{ir})w(v_{jr})W(v_{it} - v_{jt})\,dv_{ir}\,dv_{jr}\,d(v_{it} - v_{jt}),\end{aligned} \tag{3}$$

Fig. 1

Figure 1: Fig. 1

where $w(v_{ri})$ is the distribution of the projections of fall velocities onto the beam direction, and $W(v_{it} - v_{jt}) = W(\Delta v)$ is the distribution of the projections of the relative velocities of particle transport by the flow.

Fig. 1

Introduce

$$p(v_{ir}) = \frac{[\varphi(v_{ir})]^6}{\overline{[\varphi(v_{ir})]^6}}\, w(v_{ir}),$$

the distribution of the projections of gravitational fall velocities onto the beam direction, taking into account the contribution made to the radar signal by particles of different reflectivity. Here the probability density for v is multiplied by a weight characterizing the contribution to the radar signal of particles with velocity v. Thus, $p(v_{ir})$ is, in a certain sense, the distribution of the projections of fall velocities from the "radar' s point of view."

From (3) we proceed to the fluctuation spectrum of the intensity of the scattered field:

$$G(F) = \frac{1}{\pi}\int_{-\infty}^{+\infty} \overline{z(t)z(t+s)}\cos 2\pi F s\, ds =$$

$$= \frac{\{NC\,\overline{[\varphi(v_r)]^6}\}^2}{\pi}\int_0^{\infty}\int_0^{\infty}\int_{-\infty}^{+\infty} p(v_{ir})p(v_{jr})W(\Delta v_{ijt})$$

$$\times \int_{-\infty}^{+\infty} \cos 2k(v_{ir} - v_{jr} + \Delta v_{ijt})\cos 2\pi F s\, ds\, dv_{ir}\, dv_{jr}\, d(\Delta v_{ijt}) =$$

$$= \frac{\overline{A}^2}{2\pi}\int_{-\infty}^{+\infty} W(\Delta v_{ijt})\left[\int_0^{\infty} p(v_{jr})p\left(\frac{\lambda}{2}F - \Delta v_{ijt} + v_{jr}\right)\, dv_{jr}\right.$$

$$\left. + \int_0^{\infty} p(v_{ir})p\left(\frac{\lambda}{2}F + \Delta v_{ijt} - v_{ir}\right)\, dv_{ir}\right] d(\Delta v_{ijt});$$

$$\overline{A} = NC\,\overline{[\varphi(v_r)]^6}$$

is the mean signal power from the ensemble of scatterers.

Fig. 2

Figure 2: Fig. 2

By definition, $P(\Delta v) = \int_0^{\infty} p(v)\, p(v + \Delta v)\, dv$ is the distribution of the projections of the relative velocities of particle fall onto the beam direction, taking account of radar reflectivity. Introducing $P(\Delta v)$, we obtain the final expression for the fluctuation spectrum:

$$G(F) = \frac{\overline{A}^2}{2\pi} \int_{-\infty}^{+\infty} W(\Delta v) \left[P\left(\frac{\lambda}{2}F - \Delta v\right) + P\left(\frac{\lambda}{2}F + \Delta v\right) \right] d\Delta v. \tag{4}$$

From consideration of (4) it follows that the spectrum of fluctuations of the intensity of the scattered field, even in the case where the motion of the particles is determined by several factors, is expressed quite simply in terms of the distribution of the projections of the relative velocities of the scatterers onto the beam direction. Let us proceed to particular cases in which one of the factors may be neglected.

1. Gravitational fall of particles (in the absence of air currents):

$$W(\Delta v) = \delta(\Delta v), \tag{5}$$

$$G(F) = \frac{\overline{A}^2}{\pi} W\left(\frac{\lambda}{2}F\right).$$

2. Complete entrainment of particles by the air flow (the influence of gravitational velocities is negligibly small): $P(\Delta v) = \delta(\Delta v)$,

$$G(F) = \frac{\overline{A}^2}{\pi} P\left(\frac{\lambda}{2}F\right). \tag{6}$$

In both cases the spectrum of fluctuations of the intensity of the scattered field coincides in form with the distribution of the projections of the relative velocities of the scatterers onto the beam direction. Using relation (5), we obtain a connection between the fluctuation spectrum and certain characteristics of turbulent motion in the atmosphere, the inhomogeneity of the wind field, etc. Since the distribution of gravitational fall velocities of scatterers is connected with their size distribution, it is possible to study the microcharacteristics of precipitation from the fluctuation spectrum of the radar signal (6).

Fig. 2

Above, an ensemble of scatterers with identical aerodynamic properties was considered. It is of interest to consider an ensemble of dissimilar scatterers. In particular, for an ensemble of scatterers consisting of two types of particles with different laws of fall (drops and snowflakes), we obtain:

$$G(F) = \frac{1}{\pi} \left\{ \overline{A}_1^2 P_1 \left(\frac{\lambda}{2} F \right) + \overline{A}_2^2 P_2 \left(\frac{\lambda}{2} F \right) + \right.$$

$$\left. + \overline{A}_1 \overline{A}_2 \int_0^\infty P_1(v) \left[P_2 \left(v + \frac{\lambda}{2} F \right) + P_2 \left(v - \frac{\lambda}{2} F \right) \right] dv \right\}. \quad (7)$$

The relation shows that in some cases information about the phase composition of precipitation can be extracted from the spectrum of radio-echo fluctuations.

The method makes it possible to obtain information not only about the relative motions of the scatterers, but also about the distribution of the absolute velocities of the scatterers. For this purpose, a signal with a constant···

phase. Then the field strength at the receiver output can be written in the form

$$E(t) = \sum_{i=1}^{N} a_i \cos \left[\omega t - \alpha_i(t) \right] + a_0 \cos(\omega t - \alpha_0),$$

whence we obtain a series of relations for the fluctuation spectrum of such a mixed signal. For the case of complete entrainment of the particles by the air flow and neglecting their gravitational fall, we obtain

$$G(F) = \frac{1}{\pi} \left[\overline{A}^2 W \left(\frac{\lambda}{2} F \right) + 2 \overline{A}_0 \overline{A} w \left(\frac{\lambda}{2} F \right) \right], \quad (8)$$

where $w(v)$ is the distribution of the projections of the absolute velocities of the scatterers onto the direction of the radar beam. Thus, the fluctuation spectrum of the mixed signal repeats, in form, the sum of these distributions. The relation obtained is explained by Fig. 1. Figure 1a shows the distribution of the velocities of the scatterers in the meteorological object under study. Figure 1b depicts the fluctuation spectrum of the intensity of the radar signal, which, as was shown above, repeats in form the distribution of the projections of the relative velocities of the scatterers, with v and F related by

$$v = \frac{\lambda}{2} F.$$

If a signal with constant phase is mixed with the signal from the ensemble of scatterers, then the form of the spectrum is analogous to that shown in Fig. 1c.

An experimental verification of the conclusions obtained was carried out on a radar station of the 3-centimeter band. The auxiliary apparatus consisted of two units: a selector and a spectrum analyzer. The purpose of the selector was to isolate the signal from the investigated volume at a specified range and to convert it into a form convenient for analysis. From the selector output a sequence of pulses is taken, whose envelope repeats the time variation of the signal from the selected volume. For the statistical analysis a heterodyne spectrum analyzer was used. To increase the accuracy of readings and eliminate a number of errors, a mode is employed in which the spectrum recording is obtained symmetric with respect to zero frequency.

In the experiments, extensive material was accumulated on the fluctuation spectra of signals from clouds, precipitation, etc. To measure the absolute velocities of the scatterers, we mixed the signal from the ensemble of scatterers with a signal from a local object. Figure 2 presents the fluctuation spectrum of such a mixed signal. As was to be expected, a typical two-humped curve is observed, from which $W(\Delta v)$ and $w(v)$ can be determined simultaneously.

Comparison of the measurements made with meteorological data showed good agreement. However, the experimental results of these studies lie beyond the scope of the present note. We shall only indicate that the relations obtained are useful in studying both dynamic processes occurring in the atmosphere and the microstructure of precipitation.

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Note: Figure translations are in progress. See original paper for figures.