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Abstract

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On the Theory of Gas Slip Along a Solid Surface Under the Action of a Temperature Difference

The question of the "thermal" slip of a gas along a solid surface was first considered theoretically at the end of the last century by Maxwell (^{1}). He showed that, in the presence of a tangential temperature gradient $\operatorname{grad} T$, the gas begins to move along the wall, and he obtained the corresponding formula for the velocity of such slip as a function of $\operatorname{grad} T$, the viscosity of the gas η, its density ρ, and its temperature. However, as we have already noted earlier (^{2}), Maxwell' s approach to the solution of the problem was very nonrigorous. Accordingly, the value of the numerical coefficient in the formula he obtained gave rise to doubt. Nevertheless, all investigators of this problem—both theorists and experimentalists—up to the most recent time have used, for lack of anything better, precisely Maxwell' s formula. Thermal slip has also been, and is still, used to explain the thermophoretic motion of large aerosol particles with a size greater than the mean free path of the molecules of the surrounding gas. However, a number of experiments show that the theory of this phenomenon constructed on the basis of Maxwell' s formula (^{3}) does not agree with experiment, in particular with respect to the dependence of the particle velocity on the thermal conductivity of its material. Below we shall try to show how one can obtain, in a more rigorous manner, an expression for the velocity of thermal slip, using for this purpose Onsager' s principle of the thermodynamics of irreversible processes.

1. Let us consider the problem of the motion of a gas in a quasi-plane slit formed by the lateral surface of a cylinder of sufficiently large radius r, capable of rotating, and a fixed enveloping surface. The thickness of the gap is $h \ll r$. Suppose that in the gas located in the slit one can create a temperature difference. Then, when the cylinder rotates under the action of a moment M, in the presence of a temperature difference between the ends of the slit ΔT, the entropy S of the system changes according to the law:

$$\frac{dS}{dt} = v\frac{M}{rT} + \frac{I_Q \Delta T}{T^2},$$

where v is the velocity of the cylinder surface, and I_Q is the heat flux through the cross section of the slit. In accordance with the methods of the thermodynamics

Fig. 1

Figure 1: Fig. 1

of irreversible processes, the thermodynamic equations of motion may be written in the form:

$$v = L_{11}\frac{M}{rT} + L_{12}\frac{\Delta T}{T^2},$$

$$I_Q = L_{21}\frac{M}{rT} + L_{22}\frac{\Delta T}{T^2},$$

where, by Onsager' s theorem, $L_{21} = L_{12}$.

Let us consider isothermal motion ($\Delta T = 0$). The heat flux at constant temperature will be expressed in the form $I_Q = L_{21}\frac{M}{rT}$. On the other hand, when $M = 0$,

$$v\big|_{M=0} = L_{12}\frac{\Delta T}{T^2} = \frac{I_Q\big| Tr}{M}\,\frac{\Delta T}{T}.$$

Thus, having calculated the heat flux in the gas when the cylinder rotates under the action of a moment M and at constant temperature, one can find the velocity of motion of the cylinder v under the action of a temperature difference ΔT. For Couette flow of a gas in a plane slit at constant temperature we have:

$$\frac{M}{r} = \eta\frac{dv}{dx},$$

where η is the coefficient of viscosity of the gas, and $\frac{dv}{dx}$ is the velocity gradient in the gas at a sufficiently large distance from the walls (the x-axis is directed along the normal to the cylinder into the gap). Thus, the velocity of thermal motion of the cylinder is expressed in the form:

$$v\big|_{M=0} = \frac{I_Q\big|_T}{dv/dx}\,\frac{\Delta \ln T}{\eta}.$$

2. Let the surface of the cylinder be such that a fraction ε_1 of the gas molecules striking it is reflected diffusely, and $1 - \varepsilon_1$ specularly. Let, correspondingly, on the surface of the wall the coefficient of diffuse reflection be equal to ε_2. Further, let in the volume of the gas in the gap, far from the walls (we assume that the gap thickness $L \gg \lambda$, where λ is the mean free path of the gas molecules), under the influence of the rotation of the cylinder a velocity gradient be established

Fig. 1

$$\frac{dv}{dx} = \left(\frac{2kT}{m}\right)^{1/2} \frac{du}{dx},$$

where k is Boltzmann’ s constant, m is the mass of a molecule, and u is the dimensionless velocity.

Using the results of our work (^{2}), one can show that the gas in the slit is described in the near-wall region by the distribution function

$$f = f^{(0)} \left[1 + \left(\frac{a_0^+ + a_0^-}{2} + \frac{a_1^+ + a_1^-}{2} c_x\right) c_z + \left(\frac{a_0^+ - a_0^-}{2} + \frac{a_1^+ - a_1^-}{2} c_x\right) c_z \,\mathrm{sign}\, c_x\right].$$

(all notation is the same as in (^{2})).

For the region near the surface of the cylinder we have:

$$a_0^+ = \left[-\frac{B}{C} + 2\frac{x}{l} + \frac{B\varepsilon_1}{(1-\varepsilon_1)D + C} e^{-\alpha x}\right] l \frac{du}{dx},$$

$$a_0^- = \left[-\frac{B}{C} + 2\frac{x}{l} + \frac{A\varepsilon_1}{(1-\varepsilon_1)D + C} e^{-\alpha x}\right] l \frac{du}{dx},$$

$$a_1^+ = \left[-1 + \frac{C\varepsilon_1}{(1-\varepsilon_1)D + C} e^{-\alpha x}\right] l \frac{du}{dx},$$

$$a_1^- = \left[-1 + \frac{D\varepsilon_1}{(1-\varepsilon_1)D + C} e^{-\alpha x}\right] l \frac{du}{dx}.$$

Correspondingly, near the immobile wall the state of the gas is described by an analogous distribution function containing the coefficient ε_2. With the aid of the distribution function it is easy to calculate, by the usual formulas, the heat flux $I_Q\big|_T$ (the energy flux minus the enthalpy flux). We have

$$dI_Q^{(1)}\big|_T = n \left(\frac{2kT}{\pi m}\right)^{1/2} \frac{kT}{8} \frac{\varepsilon_1(C+D)}{(1-\varepsilon_1)D + C} l \frac{du}{dx} e^{-\alpha x}\, dx$$

at the surface of the cylinder. Integration with respect to x from 0 to ∞ gives

$$I_Q^{(1)}\big|_\tau = n \left(\frac{2kT}{\pi m}\right)^{1/2} \frac{kT}{8a} \frac{\varepsilon_1(C+D)}{(1-\varepsilon_1)D + C} l \frac{du}{dx}.$$

Accordingly, at the wall, taking into account the opposite sign of du/dx, we have

$$I_Q^{(2)}\Big|_\tau = -n\left(\frac{2kT}{\pi m}\right)^{1/2}\frac{kT}{8a}\,\frac{\varepsilon_2(C+D)}{(1-\varepsilon_2)D+C}\,l\frac{du}{dx}.$$

The total heat flux per unit length of the cylinder will be:

$$I_Q\Big|_\tau = I_Q^{(1)}\Big|_\tau + I_Q^{(2)}\Big|_\tau =$$

$$= \frac{nkT}{8}\left(\frac{2kT}{\pi m}\right)^{1/2}(C+D)\frac{l}{a}\frac{du}{dx}\left[\frac{\varepsilon_1}{(1-\varepsilon_1)D+C} - \frac{\varepsilon_2}{(1-\varepsilon_2)D+C}\right].$$

Thus, for the rotational velocity of the cylinder under the influence of a temperature difference, we have (substituting $a = 7.56/\lambda$)

$$v = \frac{\eta}{\rho}\Delta\ln T\,\frac{4(C+D)}{5\pi\cdot 7.56}\left[\frac{\varepsilon_1}{(1-\varepsilon_1)D+C} - \frac{\varepsilon_2}{(1-\varepsilon_2)D+C}\right].$$

From the formula obtained it is seen that the rotational velocity of the cylinder is the difference of quantities, each depending only on the properties (ε_1 and ε_2) of one of the two surfaces (the cylinder and the fixed enclosure), and not on their mutual distance. This is physically possible only in the case where v is the difference of two velocity jumps—two velocities of thermal slip of the gas relative to the solid surfaces. In this case the velocity of thermal slip of the gas along the surface must have the form:

$$v_c = \frac{\eta}{\rho}\Delta\ln T\,\frac{4}{5\pi\cdot 7.56}\,\frac{\varepsilon_1(C+D)}{(1-\varepsilon_1)D+C},$$

or, for $\varepsilon_1 = 1$,

$$v_c = \frac{\eta}{\rho}\Delta\ln T\,\frac{4}{5\pi\cdot 7.56}\left(1+\frac{D}{C}\right).$$

Substituting here the values of D and C, calculated in (^{2}), we obtain

$$v_c = 0.0218\,\frac{\eta}{\rho}\,\Delta\ln T.$$

Comparison of the expression obtained with Maxwell' s formula for this case shows that they differ only in the coefficients, the coefficient in our formula being smaller by approximately a factor of 35.

The calculation carried out permits one to suppose that the motion of aerosol particles in a temperature field is determined, in the main, not by the slip effect, but by some other mechanism.

An indication of this is also the fact that the thermophoretic velocity of small aerosol particles, calculated by us (4) for the case of specular reflection, obviously not connected in any way with thermal slip at macrosurfaces, coincides with the Maxwellian value of the velocity of "thermal slip."

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Note: Figure translations are in progress. See original paper for figures.

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