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Abstract

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THE TRANSVERSE REFRACTIVE INDEX OF A PLASMA NEAR CYCLOTRON FREQUENCIES AND THEIR HARMONICS

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Oscillations are considered of a homogeneous unbounded plasma placed in a constant homogeneous magnetic field H_0, directed along the z axis. The wave vector $\mathbf{k}$ is directed along the x axis. The general dispersion equation for this case is well known (see, for example, ([1])); however, until now there has been no clarity on the question of the behavior of the refractive index near cyclotron frequencies and their harmonics.

From the solution of the kinetic equation for charged particles distributed in velocity space according to Maxwell, and of Maxwell' s equations for an electromagnetic wave, we obtain the following values of the components of the dielectric-permittivity tensor:

$$\varepsilon_{xx} = 1 - \sum_\alpha \frac{\Omega_{p\alpha}^2}{\Omega_{e\alpha}^2} \frac{e^{-r_\alpha}}{r_\alpha} \sum_{m=-\infty}^{\infty} \frac{m^2 I_m(r_\alpha)}{\omega^2/\Omega_{e\alpha}^2 - m^2},$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = i \sum_\alpha \frac{\Omega_{p\alpha}^2}{\omega\Omega_{e\alpha}} e^{-r_\alpha} \sum_{m=-\infty}^{\infty} \frac{m^2 \left[I_m'(r_\alpha) - I_m(r_\alpha)\right]}{\omega^2/\Omega_{e\alpha}^2 - m^2},$$

$$\varepsilon_{yy} = 1 - \sum_\alpha \frac{\Omega_{p\alpha}^2}{\Omega_{e\alpha}^2} \frac{e^{-r_\alpha}}{r_\alpha} \sum_{m=-\infty}^{\infty} \frac{m^2 I_m(r_\alpha) + 2r_\alpha^2 \left[I_m(r_\alpha) - I_m'(r_\alpha)\right]}{\omega^2/\Omega_{e\alpha}^2 - m^2},$$

$$\varepsilon_{zz} = 1 - \sum_\alpha \frac{\Omega_{p\alpha}^2}{\Omega_{e\alpha}^2} \sum_{m=-\infty}^{\infty} e^{-r_\alpha} \frac{I_m(r_\alpha)}{\omega^2/\Omega_{e\alpha}^2 - m^2},$$

where summation over α is summation over the species of charged particles, $I_m(r)$ is the Bessel function of the first kind of imaginary argument, $r_\alpha = k^2 v_{T\alpha}^2 / 2\Omega_{e\alpha}^2$, $v_{T\alpha}^2 = \varkappa T_\alpha / m_\alpha$, $\Omega_{e\alpha} = eH_0/m_\alpha c$, $k = |\mathbf{k}|$, $\Omega_{p\alpha}^2 = 4\pi e^2 n / m_\alpha$; the remaining components ε_{ik} are equal to zero. From these expressions one can draw the following conclusions about the behavior of r (or N^2) as a function of frequency near $m\Omega_{e\alpha}$ (it is assumed that $\Omega_p^2 \gg \Omega_e^2$).

Fig. 1 and Fig. 2: schematic plots of N^2 versus ω, with resonances marked at $\Omega_i, 2\Omega_i, 3\Omega_i$ and $\Omega_e, 2\Omega_e, 3\Omega_e$, respectively.

Figure 1: Fig. 1 and Fig. 2: schematic plots of N^2 versus ω, with resonances marked at $\Omega_i, 2\Omega_i, 3\Omega_i$ and $\Omega_e, 2\Omega_e, 3\Omega_e$, respectively.

Extraordinary wave (the electric field of the wave is perpendicular to the constant magnetic field H_0)

$$N^2 = \varepsilon_{yy} + \frac{\varepsilon_{xy}^2}{\varepsilon_{xx}}.$$

The thermal motion of the electrons has little effect on the behavior of the refractive index near the ion cyclotron resonance and its harmonics; therefore, when considering the behavior of N^2 near the first resonances, the electrons may be regarded as cold and the electron terms in the expressions may be neglected.

for ε_{ik}. The behavior of the resulting refractive index is shown in Fig. 1. The width of the regions in which the behavior of the refractive index differs sharply from the behavior of N^2 in a cold plasma is of the order of the ratio of the gas pressure to the magnetic pressure at the first two resonances and decreases as $(p_{\text{gas}}/p_{\text{magn}})^{m-1}$ at the subsequent ones. It is superfluous to consider very high harmonics of the ion cyclotron frequency, since the dissipative processes not taken into account in our treatment (the relativistic Doppler effect, collisions) lead to a smearing of the resonance peaks and to the formation of a certain average refractive index and absorption.

In considering the behavior of the refractive index near the electron cyclotron resonance and its harmonics ($\omega^2 \ll \Omega_{pe}^2$), one cannot simply discard the ion terms in ε_{ik}. These terms will affect the width of those regions in which the behavior of N^2 differs sharply from the behavior in a cold plasma.

Fig. 1. Behavior of N^2—the refractive index of the extraordinary wave—near the ion cyclotron frequency and its first harmonics in a warm plasma

Fig. 2. Behavior of N^2—the refractive index of the extraordinary wave—near the first harmonics of the electron cyclotron frequency in a warm plasma

Finally, one obtains that N^2, as a function of frequency, behaves approximately as shown in Fig. 2. It is seen that near $m\Omega_e$, N^2 can have 1, 2, 3, 4, or no real values. In the regions where there are no real values, there are two complex-conjugate roots. The existence of regions in which N^2 is complex in the absence of dissipative mechanisms had already been noted in the literature for the case of coincidence of $m\Omega_e$ and $\sqrt{\Omega_e^2 + \Omega_p^2}$ ([2]). It turns out that regions of complex values of N^2 exist near all harmonics of the cyclotron frequency in those frequency regions where, when thermal motion is neglected, N^2 was negative. (It should be noted that a similar situation also obtains near the harmonics of the ion frequency in that frequency region where, when thermal motion is neglected, $N^2 < 0$; consideration of this region was abandoned earlier.)

Another essential feature of the behavior of N^2 near $m\Omega_e$ when thermal motion is taken into account is the presence of pass bands ($N^2 > 0$) in that frequency region where, when thermal motion is neglected, N^2 was negative (see Fig. 2).

The relative width of the regions near the harmonics of the electron cyclotron frequency in which the behavior of N^2 differs sharply from the behavior in a cold plasma (with allowance for the thermal motion of both electrons and ions) remains the same as near the harmonics of the ion cyclotron frequency. With increasing

frequency, but with the condition $\omega^2 \ll \Omega_e^2$ satisfied, the picture shown in Fig. 2 is repeated.

A similar consideration of the ordinary wave (the electric field of the wave parallel to the external magnetic field) was carried out in Ref. [3]. The authors note the same essential features as those indicated above for the extraordinary wave.

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Note: Figure translations are in progress. See original paper for figures.