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1961

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Abstract

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MATHEMATICAL PHYSICS

S. P. LOMNEV

A METHOD FOR SOLVING THE MANY-BODY PROBLEM ON HIGH-SPEED COMPUTING MACHINES FOR THE CASE OF CHARGED PARTICLES

(Presented by Academician A. A. Dorodnitsyn on 30 VI 1961)

The relativistic equations of motion of M charged interacting particles, with allowance for an external field, have the form

$$\left\{ \frac{d\mathbf{P}_m}{dt} = \mathbf{A}_m(t, \mathbf{R}_m, \dot{\mathbf{R}}_m) + \sum_{k=1}^M \mathbf{a}_{mk} \right\}_{m=1}^M, \quad (1)$$

where \mathbf{P}_m is the momentum vector of the particle; \mathbf{R}_m is the vector of its position; $\dot{\mathbf{R}}_m$ is its velocity; \mathbf{A}_m is the external field; \mathbf{a}_{mk} is the interaction force, expressed through the Liénard-Wiechert potential ⁽¹⁾

$$\mathbf{a}_{mk} = e_m \left\{ \vec{\varepsilon}_k + \frac{1}{c} [\dot{\mathbf{R}}_k \mathbf{h}_k] \right\},$$

$$\vec{\varepsilon}_k = e_k \left\{ \frac{1 - \dot{\mathbf{R}}_k^2/c^2}{(R_{km} - \mathbf{R}_{km} \cdot \dot{\mathbf{R}}_k/c)^3} \left(\mathbf{R}_{km} - \frac{\dot{\mathbf{R}}_k R_{km}}{c} \right) + \frac{[\mathbf{R}_{km} ((\mathbf{R}_{km} - \dot{\mathbf{R}}_k R_{km}/c) \ddot{\mathbf{R}}_k)]}{c^2 (R_{km} - \mathbf{R}_{km} \dot{\mathbf{R}}_k/c)^3} \right\},$$

$$\mathbf{h}_k = \frac{1}{R_{km}} [\mathbf{R}_{km} \cdot \vec{\varepsilon}_k]. \quad (2)$$

Here e_m is the charge of the particle; c is the speed of light; $\mathbf{R}_{km} = \mathbf{R}_k - \mathbf{R}_m$; all quantities in (2) are taken for $\tau_{km} = t - R_{km}/c$.

If at $t = 0$ the initial conditions are specified,

$$\mathbf{R}_m = \mathbf{R}_{m0}, \quad \dot{\mathbf{R}}_m = \dot{\mathbf{R}}_{m0}, \quad (3)$$

then the motion of the ensemble of particles $m = 1, 2, \dots, M$ is completely determined by the solution of the system (1), (2), (3):

$$\begin{aligned}\mathbf{R}_m(t) &= \mathbf{f}(t; \mathbf{R}_{10}, \dots, \mathbf{R}_{M0}; \dot{\mathbf{R}}_{10}, \dots, \dot{\mathbf{R}}_{M0}), \\ \dot{\mathbf{R}}_m(t) &= \mathbf{g}(t; \mathbf{R}_{10}, \dots, \mathbf{R}_{M0}; \dot{\mathbf{R}}_{10}, \dots, \dot{\mathbf{R}}_{M0}).\end{aligned}\tag{4}$$

In view of the exceptional complexity of the system (1), (2), for large M its solution by analytical methods does not appear possible. This circumstance forced researchers to turn to a statistical concept in the many-body problem. Now, when high-speed computing machines with large random-access memory are being created, there has appeared a fundamental possibility of returning to the original system of many-body equations, the solution of which gives a complete description of the dynamical evolution of an ensemble of M interacting particles.

Because of the limited random-access memory of the machine that we had at our disposal, the largest number of equations (1) that could be solved did not exceed 100. Therefore we were compelled to reduce the number of initial equations. For this purpose, the region V_0 in which the charges exist at $t = 0$ is divided into small volumes ΔV_k (they may be equal), and each charge contained in ΔV_k is concentrated at its center. It is regarded as a new enlarged particle. In all one obtains $N \ll M$ “particles,” the motion

which is determined by the same system of equations (1), (2) with the corresponding indices. As initial conditions one takes averages over the particles inside ΔV_k ($k = 1, 2, \dots, N$). It is assumed here that during the time $0 < t < T$ the actual particles in the volumes ΔV_k behave as a single whole, which is admissible for finite T (the particle transit time) and sufficiently small ΔV_k , when long-range interactions prevail over short-range ones inside ΔV_k . The admissible size depends on the coefficients of (1) and on the dispersion of the initial velocities in ΔV_k . It is determined by the convergence of (4) under an additional decrease of ΔV_k .

For the system of equations obtained there exist a number of numerical methods of solution. In the case described below we used the Runge–Kutta method with automatic choice of the step in t . The second derivatives were taken into account in computing the sum on the right by the method of successive iterations at each integration step. As the zero approximation, the expression for the right-hand side of (1) with $a_{mk} = 0$ was taken.

As an example, let us consider the problem of the motion of a beam of charged particles in a linear electron accelerator operating on the principle of autophasing [2]. In this case the phenomenon of particle interaction plays an essential role, since it makes it impossible to obtain large currents in the beam because of phase oscillations of the particles, which leads to broadening of the beam and to the loss of some particles on the accelerator walls.

We solve the problem in the cylindrical coordinate system r, z, θ . Taking into account that the component of the electric field $E_\theta = 0$ and that the initial conditions do not depend on θ , we obtain a solution independent of θ . Therefore we

shall consider a two-dimensional problem for the coordinates r and z , carrying out integration with respect to θ , i.e., writing the equations of motion for geometric manifolds with coordinates $r_{s,l}; z_{s,l}; 0 \leq \theta \leq 2\pi$, which are rings, where s and l are the indices enumerating the rings along the r and z axes, respectively. Each ring (this is one enlarged particle) is described by the system of equations

$$\begin{aligned} \ddot{r}_{s,l} &= A_0(1 - \dot{z}_{s,l}^2)^{1/2} \left\{ \alpha_{s,l} H^* - \frac{r_{s,l}}{2} \cos \varphi_{s,l} [\beta_0 - \dot{z}_{s,l} k] \right. \\ &\quad \left. - \dot{r}_{s,l} \dot{z}_{s,l} \sin \varphi_{s,l} + \varepsilon_{r,s,l} - \dot{z}_{s,l} \dot{r}_{s,l} \varepsilon_{z,s,l} \right\} + \frac{\dot{\alpha}_{s,l}^2}{r_{s,l}}, \\ \ddot{z}_{s,l} &= A_0(1 - \dot{z}_{s,l}^2)^{3/2} \{ \sin \varphi_{s,l} + \varepsilon_{z,s,l} \}, \\ \dot{\varphi}_{s,l} &= \beta_0 \dot{z}_{s,l} - k, \quad l = 0, 1, \dots, L; \quad s = 0, 1, \dots, S, \quad LS = N; \end{aligned} \quad (5)$$

differentiation has been performed with respect to ct ,

$$\begin{aligned} A_0 &= -\frac{e_m}{m_0 c^2} \varepsilon_0(z), \quad H^* = \frac{z_0 H_z(z)}{\varepsilon_0(z)}, \\ \alpha_{s,l} &= \frac{(1 - \dot{z}_{s,l}^2)^{1/2} A_{\theta,s,l}}{(r_{s,l}^2 + A_{\theta,s,l}^2)^{1/2}}, \quad A_{\theta,s,l} = \int A'_{\theta,s,l} c dt, \\ A'_{\theta,s,l} &= -\dot{r}_{s,l} H^*; \end{aligned}$$

e, m_0 are the charge and mass of the electron; c is the speed of light; $\varepsilon_0(z)$ is the amplitude of the accelerating field strength of the traveling wave; $H_z(z)$ is the strength of the focusing magnetic field; $k = \omega_0/c$ is the wave vector of free space; $\beta_0 = k/\beta_B$; β_B is the phase velocity of the wave; φ is the phase of the particle relative to the phase of the wave; z_0 is the impedance of free space.

The interaction of the “rings” is taken into account by the terms $\varepsilon_{z,s,l}, \varepsilon_{r,s,l}$, which are projections of $\vec{\varepsilon}_{s,l}$ onto the corresponding axes, where

$$\begin{aligned} \vec{\varepsilon}_{s,l} &= \frac{1}{\varepsilon_0} \sum_{i=1}^S \sum_{p=1}^L \int_0^{2\pi} \left\{ \frac{e_{i,p} R_{i,p}}{\eta_{i,p}^3} (\mathbf{R}_{i,p}/R_{i,p} - \mathbf{R}_{i,p}) [1 - \dot{\mathbf{R}}_{i,p}^2 + (\mathbf{R}_{i,p} \dot{\mathbf{R}}_{i,p})] - \frac{e_{i,p} R_{i,p} \ddot{\mathbf{R}}_{i,p}}{\eta_{i,p}^2} \right\}_{t-R_{i,p}/c} d\theta, \\ \eta_{i,p} &= [(z_{s,l} - z_{i,p})^2 + (1 - \dot{z}_{i,p}^2)(r_{s,l} - r_{i,p})^2]^{1/2}; \end{aligned} \quad (6)$$

$\mathbf{R}_{i,p}$ is the vector from the point of observation to the charge.

It follows from direct calculations that, with a sufficient degree of accuracy, all quantities in (6) may be referred to the time instant t , and the terms with $\ddot{\mathbf{R}}$ may be discarded.

Fig. 1. Example of the dependence I_{capt} on I_{inj} for an accelerator built at the Moscow Engineering-Physics Institute, at 2 MeV.

Figure 1: Fig. 1. Example of the dependence I_{capt} on I_{inj} for an accelerator built at the Moscow Engineering-Physics Institute, at 2 MeV.

As initial conditions we take:

$$\begin{aligned} t_0 &= 0, & \dot{z}_{0,s,l} &= \text{const}, & z_{0,s,l} &= 0, \\ \varphi_{0,s,l} &= \frac{2\pi}{L}l, & \dot{r}_{0,s,l} &= 0, & r_{0,s,l} &= \frac{r_{0\text{max}}}{S}s. \end{aligned} \quad (7)$$

The system (5), (6), (7) was solved on the BESM of the Computing Center of the Academy of Sciences of the USSR for various values of the focusing magnetic field, of the current of the injected particles, and of various types of accelerating waveguides.

As the calculation showed, for small currents (less than 0.1 A), the effect of repulsion in the radial direction may be estimated from the behavior of the boundary cell (the motion of the others is taken into account without interaction). It then turned out that particles near the axis perform small oscillations, and therefore the central core with $r_0 < \frac{2}{3}r_{0\text{max}}$ may be regarded as unchanged in the radial direction. In considering the first expansion of the beam, only a small region near the phase of the equilibrium particle $\varphi_0 = \pi$ (of order 1 rad.) has a substantial influence. The remaining part of the bunch changes little in the radial direction.

Fig. 1. Example of the dependence $I_{\text{capt}}(I_{\text{inj}})$ for an accelerator built at the Moscow Engineering-Physics Institute, at 2 MeV.

For currents of the order of $0.1 \text{ A} < I_{\text{inj}} < 1 \text{ A}$, with a solution accuracy of 1%, a mesh with $S = 4$, $L = 16$ proved sufficient. However, in order to estimate particle losses with the same degree of accuracy, the number of subdivisions in r must be increased to 8.

The effect of the expansion of a beam of particles in the radial direction is stronger than the repulsion in the longitudinal direction ⁽³⁾. It cannot be compensated by increasing H_z .

With increasing I_{inj} , particle losses on the walls grow (Fig. 1). With increasing $d\beta_B/dz$ and $d\varepsilon_0/dz$, this effect is enhanced; therefore, for obtaining large I_{capt} , the most promising accelerators will apparently be those with minimal phase oscillations, which is realized when $\beta_B = \text{const}$ and $\varepsilon_0 = \text{const}$, with the largest possible ε_0 .

It is obvious that an analogous treatment is suitable for a linear proton accelerator, a synchrocyclotron, an annular phasotron, and a cyclotron with a spatially varying magnetic field.

I consider it my duty to express my gratitude to A. A. Abramov and G. A. Tyagunov.

Computing Center
Academy of Sciences of the USSR

Received
29 VI 1961

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