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Abstract

Full Text

MATHEMATICS

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ON FRAMES OF SUBMANIFOLDS IN THE THEORY OF PAIRS OF COMPLEXES IN P_3

(Presented by Academician P. S. Aleksandrov on 17 III 1961)

In the present note the method of frames of submanifolds ⁽¹⁾ is applied to the theory of pairs of complexes.

1. Let us consider, in three-dimensional projective space, a pair of complexes described by the corresponding rays A_1A_2 and A_3A_4 . The points of intersection of these rays with the directrices of a linear congruence L , belonging simultaneously to two linear complexes, each of which touches ⁽⁴⁾ one complex of the pair and passes through the corresponding ray of the other complex of this pair, are called k -flecnodal points. To obtain the canonical frame of an arbitrary submanifold, i.e., of a pair of nonholonomic congruences ⁽²⁾ belonging to the given pair of complexes, a semicanonical frame ⁽¹⁾ of the pair of complexes is constructed. Placing the vertices of this frame A_i at the points harmonically dividing the k -flecnodal points of the corresponding rays of the pair of complexes, and carrying out invariant normalization, we obtain the derivational formulas:

$$dA_i = \omega_i^k A_k = (\alpha_i^k \omega_1^3 + \beta_i^k \omega_2^4 + \gamma_i^k \omega_3^1) A_k \quad (i, k = 1, 2, 3, 4), \quad (1)$$

where

$$\alpha_3^3 = \beta_2^4 = \gamma_3^1 = 1, \quad \beta_1^3 = \gamma_1^3 = \alpha_2^4 = \gamma_2^4 = \alpha_3^1 = \beta_3^1 = 0,$$

$$\alpha_1^1 + \alpha_2^2 + \alpha_3^3 + \alpha_4^4 = \beta_1^1 + \beta_2^2 + \beta_3^3 + \beta_4^4 = \gamma_1^1 + \gamma_2^2 + \gamma_3^3 + \gamma_4^4 = 0,$$

$$\gamma_1^4 = -\gamma_2^3 = 1, \quad \alpha_1^4 + \alpha_2^3 = \beta_1^4 + \beta_2^3,$$

$$\alpha_4^2(\gamma_3^2 + \gamma_4^1) = (\gamma_2^4 + 1)(\alpha_3^2 + \alpha_4^1), \quad \beta_4^2(\gamma_3^2 + \gamma_4^1) = (\gamma_4^2 + 1)(\beta_3^2 + \beta_4^1).$$

In constructing the semicanonical frame of a pair of complexes, the following are excluded from consideration: a) pairs T of complexes ^(6,7); b) parabolic pairs of complexes, i.e., pairs of complexes with a parabolic linear congruence L ; c)

pairs of special complexes. Exterior differentiation of the formulas leads to a system of 12 independent quadratic equations containing 29 unknown functions and defining the pair of complexes assigned to an arbitrary pair of nonholonomic congruences, with arbitrariness in 7 functions of 3 arguments.

2. Considering an arbitrary pair of nonholonomic congruences $\omega_i^k = 0$ ($i = 1, \dots, 4; |i - k| = 2$), belonging to the given pair of complexes, we obtain that the semicanonical frame of the pair of complexes is the canonical frame of this pair of nonholonomic congruences, since the point A_i on the ray of one complex describes a focal nonholonomic surface, while the point A_j ($|k - j| = 1$) lies on the corresponding focal plane.

If one of the pairs of nonholonomic congruences $\omega_i^k = 0$ ($|i - k| = 2$) is given, then the remaining ones are geometrically determined, and consequently the pairs of ruled surfaces belonging simultaneously to each two of these pairs of nonholonomic congruences are completely determined geometrically. Moreover, by assigning a coordinate pair of ruled surfaces, be-

longing simultaneously to any two of the pairs of nonholonomic congruences $\omega_i^k = 0$ ($|i - k| = 2$), are completely determined, as are the remaining pairs of nonholonomic congruences. In order to obtain the derivation formulas of the canonical frame of an arbitrary pair of nonholonomic congruences $\omega_1^3 = 0$, it is sufficient in formulas (1) to put

$$\omega_1^3 = 0, \quad (\beta_i^k)_{\omega_1^3=0} = y_i^k, \quad (\gamma_i^k)_{\omega_1^3=0} = z_i^k. \quad (2)$$

Since the frame is now geometrically determined, the coefficients y_i^k and z_i^k in the derivation formulas (1), under condition (2), are projective invariants of the pair of nonholonomic congruences. Among the functions y_i^k and z_i^k there are 21 independent ones. The coordinate pair of ruled surfaces $\omega_2^4 = 0$ ($\omega_3^1 = 0$) of the given pair of nonholonomic congruences is characterized by the fact that the tangent plane at the point A_2 (A_3) of the ruled surface described by the ray A_1A_2 (A_3A_4) intersects the ray A_3A_4 (A_1A_2) at the point A_3 (A_2).

3. Various relations between the invariants y_i^k and z_i^k distinguish classes of pairs of nonholonomic congruences belonging to the given pair of complexes. A pair h of nonholonomic congruences has the natural equations $y_4^2 = z_4^2 = 0$ and is characterized by each of the following properties: 1) the vertex A_4 is the focus of a ray of the nonholonomic congruence (A_3A_4), and the point A_1 lies on the corresponding focal plane ($z_3^2y_4^1 - y_3^2z_4^1 \neq 0$); 2) the points A_1 and A_4 are quasiflecnodal ⁽³⁾ points of the corresponding rays of any pair of ruled surfaces belonging to h . The coordinate pair of ruled surfaces $\omega_2^4 = 0$ ($\omega_3^1 = 0$) of the pair h , in addition to the properties noted above, is also characterized by the fact that the tangent plane at the k -flecnodal point $A_1 \pm A_2$ ($A_3 \pm A_4$) of the ruled surface described by the ray A_1A_2 (A_3A_4) intersects the ray A_3A_4 (A_1A_2) at the k -flecnodal point $A_3 \mp A_4$ ($A_1 \mp A_2$). The pair of nonholonomic congruences

$y_3^2 = y_4^1 z_3^2 - y_4^2 = 0$ is characterized by the fact that the nonholonomic congruence $(A_3 A_4)$ is parabolic with focal nonholonomic surface described by the point A_3 . The pair of nonholonomic congruences $y_4^1 = y_4^2 z_3^2 - y_3^2 z_4^2 = 0$ is characterized by the fact that the focal planes at the foci of a ray of the nonholonomic congruence $(A_3 A_4)$ intersect the ray $A_1 A_2$ at the points A_1 and A_2 . The pair of nonholonomic congruences

$$(y_4^4 - y_2^3)(z_4^2 - z_3^2 z_4^1) - y_4^2 + y_3^2 z_4^1 + z_3^2 y_4^1 = y_2^3 y_4^1 - y_4^1 y_2^3 (z_4^2 - z_3^2 z_4^1) = 0$$

is characterized by the fact that the torsos of the nonholonomic congruences $(A_1 A_2)$ and $(A_3 A_4)$ correspond. The pair of nonholonomic congruences $y_3^2 = z_4^2 y_4^1 - y_4^2 z_4^1 = 0$ is characterized by the fact that the points A_3 and A_4 describe the focal nonholonomic surfaces of the nonholonomic congruence $(A_3 A_4)$. The pair of nonholonomic congruences

$$y_4^2 - y_4^1 + y_3^2 = z_4^2 - z_4^1 + z_3^2 + 1 = 0$$

$$(y_4^2 - y_3^2 + y_4^1 = z_4^2 - z_3^2 + z_4^1 + 1 = 0)$$

is geometrically characterized by the fact that the k -flecnodal point $A_3 - A_4$ ($A_3 + A_4$) describes the focal nonholonomic surface of the parabolic nonholonomic congruence $(A_3 A_4)$.

4. The presence of submanifolds of a special type characterizes two most important pairs of complexes. The pair R_1 of complexes, containing the submanifold $z_4^2 = y_4^2 = z_4^1 + z_3^2 + y_4^1 + y_3^2 = 0$, is geometrically characterized by each of the following properties: 1) the pair h of nonholonomic congruences forms a pair T (the pair T of nonholonomic congruences is defined in the same way as the pair T of holonomic congruences ⁽⁶⁾); 2) in the pair h there are at least two stratifiable pairs of ruled surfaces; 3) the foci of the corresponding rays of the pair h are quasiflecnodal points of any pair of ruled surfaces of this pair.

A pair R_2 of complexes containing the submanifold $y_4^2 = z_4^2 = z_4^1 + y_3^2 = z_3^2 + y_4^1 = 0$ is characterized by each of the following properties: 1) a pair h of nonholonomic congruences of a stratification on both sides (a pair of nonholonomic congruences stratified on one or both sides is defined in the same way as a stratified pair of holonomic congruences ⁽⁶⁾); 2) the compound ratio ω (the compound ratio of the tangents to four curves on any focal nonholonomic surface of the pair h of nonholonomic congruences), corresponding to the coordinate pairs of ruled surfaces and torsos of one nonholonomic congruence of the pair h , is equal to the ratio corresponding to the coordinate pairs of ruled surfaces and torsos of the other nonholonomic congruence of the pair h ; 3) the coordinate pairs of ruled surfaces of the pair h harmonically divide (i.e., $\omega = -1$) the stratified pairs of ruled surfaces of the same pair of nonholonomic congruences; 4) the stratified

pairs of ruled surfaces of the pair h harmonically divide the pairs of ruled surfaces conjugate both in one and in the other nonholonomic congruences of the pair h (conjugate ruled surfaces of a nonholonomic congruence are defined in the same way as ruled surfaces of a holonomic congruence conjugate in the sense of Sannia⁽⁵⁾); 5) the stratified pairs of ruled surfaces of the pair h correspond to the asymptotic lines on all stratifying nonholonomic surfaces of both families. Pairs R_2 of complexes form a subclass of the class of pairs R_1 of complexes.

5. In the usual way we find that pairs R_1 and R_2 of complexes exist and are determined with arbitrariness in 4 and 3 functions of 3 arguments, respectively.

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