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Abstract

Full Text

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ON EQUIVALENT TRANSFORMATIONS OF CONTACT-RECTIFIER CIRCUITS

(Presented by Academician M. V. Keldysh on 9 VIII 1960)

In the note ([1]), equivalent transformations of contact circuits were considered by replacing subcircuits with equivalent ones (for the definition of a contact circuit, see, for example, ([2])). A pair of equivalent circuits A and B specifies a rule $A \leftrightarrow B$, according to which, in any circuit, a subcircuit obtained from A by some renaming of letters may be replaced by the subcircuit obtained from B by the same renaming of letters, and conversely. In the present note, from a similar point of view, the class of contact-rectifier circuits is considered.

Fig. 1. Rules of the system Γ. Poles are denoted by circles; the correspondence between poles is established by their numbering; the circuits in rule 1 are poleless; the right-hand side of 1 is the empty circuit.

It turns out that Theorem 3 of ([1]), according to which there does not exist a finite complete* system of transformation rules for contact circuits**, does not remain valid when passing to contact-rectifier circuits.

We shall call two contact-rectifier circuits **equivalent** (under a given one-to-one correspondence between the poles) if, for every ordered pair (a, b) of poles of one of these circuits, the conductance in the second circuit from the pole corresponding to a to the pole corresponding to b is equal to the conductance in the first circuit from a to b. Further, we shall call two circuits **equivalent with respect to a system of rules** α, or simply α-equivalent, if they can be obtained from one another by applying the rules of the system α.

Let Γ be the system of rules 1–6 (Fig. 1).

* That is, one allowing any circuit to be transformed into any circuit equivalent to it.

Figure 2: derivable rules 7–13

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** If, in applying the rules, only renaming of letters is allowed.

Theorem. Any two equivalent contact-valve circuits are Γ-equivalent.

Let X_n denote the set $\{x_1, \dots, x_n\}$ of distinct variables. We shall call an **elementary unoriented X_n-chain** a subcircuit consisting of n sequentially connected contacts

$$x_1^{\sigma_1},\ x_2^{\sigma_2}, \dots,\ x_n^{\sigma_n} \qquad (\sigma_i = 0 \text{ or } 1).$$

By adjoining a valve to these contacts in series (at any place), we obtain an **elementary oriented** X_n-chain. Elementary chains obtained from one another by a permutation of contacts and valves* will not be distinguished**.

Fig. 2. "Derivable" rules 7–13. The letter I denotes an arbitrary conjunction of the form

$$x_1^{\sigma_1} \cdots x_k^{\sigma_k},$$

and $I_1^m, I_2^m, \dots, I_{2^n}^m$ are all possible distinct conjunctions of the form

$$x_1^{\sigma_1} \cdots x_n^{\sigma_n}.$$

In the right-hand part of rule 12, each "input" pole is directly connected by an elementary oriented chain with each "output" pole.

Elementary chains will be denoted by conjunctions

$$x_1^{\sigma_1} \cdots x_n^{\sigma_n},$$

written on the edges (or, for oriented chains, on oriented edges). Let A be an X_n-circuit, i.e. a circuit over the alphabet

$$\{x_1, \dots, x_n, \bar{x}_1, \dots, \bar{x}_n\}.$$

Let $a_1, \dots, a_k$ be the poles of the circuit A. Consider the X_n-circuit B with poles $a_1, \dots, a_k$ having the following form: for each ordered pair (a_i, a_j) of distinct poles and each set $(\sigma_1, \dots, \sigma_n)$ of zeros and ones such that the conductivity from a_i to a_j is equal to 1 when

$$x_1 = \sigma_1, \dots, x_n = \sigma_n,$$

the circuit B contains an elementary oriented X_n-chain going from a_i to a_j*** and containing the contacts

$$x_1^{\sigma_1}, \dots, x_n^{\sigma_n}.$$

We shall call the circuit B the **canonical form** of the X_n-circuit A. B is equivalent to A; therefore the theorem as formulated follows from the following lemma.

Fig. 3

Figure 3: Fig. 3

Lemma 1. Every X_n-circuit A can, by transformations of the system Γ, be brought to the canonical form B.

We shall call an **elementary X_n-circuit** an X_n-circuit obtained from some network with oriented and unoriented edges by assigning to each edge of the network one and the same conjunction

$$x_1^{\sigma_1} \cdots x_n^{\sigma_n}.$$

Lemma 1 on reduction to canonical form follows from Lemmas 2 and 3.

* Without changing the orientation of the valve.
** Such chains are transformed into one another by rules 3_1 and 3_2.
*** That is, the ends of the chain are a_i and a_j, and its valve is oriented from a_i to a_j.

Lemma 2. Every X_n-circuit A is Γ-equivalent to some circuit B formed by gluing together, at the poles, several elementary X_n-circuits (with the same poles as A).

Lemma 3. Every elementary X_n-circuit can be brought, by transformations of system Γ, to canonical form.

For the proof of these lemmas we shall need certain rules that are consequences of the rules Γ. These are rules 7-13 (Fig. 2). We omit the proofs of the "derivability" of these rules from the rules Γ.*

Proof of Lemma 2. Replace each contact and valve of the given circuit A, by rules $7_1, 7_2$, with parallel-connected elementary X_n-chains. Let a be a vertex of the obtained circuit A', not a pole, at which elementary chains corresponding to different conjunctions meet. Attach to a, by rule 8, all 2^n elementary unoriented X_n-chains (see the example in Fig. 3a, where $n = 2$). Then each elementary chain of the circuit A' that has the point a as one of its ends is "thrown over," by rules $9_1, 9_2$, or 9_3, to the other end of the corresponding attached chain. The remaining "star" with center at a is removed by rule 10. In the resulting circuit A'' the number of such internal vertices that are ends of chains with nonidentical conjunctions is one less than in A'. Therefore, carrying out the same process a sufficiently large number of times, we obtain the required circuit B.

Fig. 3

Proof of Lemma 3. Let an elementary X_n-circuit be given, and let a be a point of contact of elementary X_n-chains of this circuit, not a pole. Consider the chains forming the star of the vertex a. By successively applying rules $11_1, 11_2, 11_3, 11_4$, one can eliminate parallel chains from the star. After this: if all chains in the star are oriented, we replace it by rule 12; if there is an unoriented chain, then, applying rules $9_1, 9_2$, and 9_3, we "throw over" from the star all the remaining elementary chains (Fig. 3b), after which the remaining chain

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* The complete proofs of the theorem of the present note and of the theorems from (1) will be published in the collection *Problems of Cybernetics*, issue 5.

is eliminated by means of 8. Repeating such a process a sufficient number of times, we obtain an elementary X_n-circuit in which the elementary circuits meet only at the poles. To obtain the canonical form, it remains only to: eliminate parallel elementary circuits; then replace each unoriented circuit, according to rule 11_2, by a pair of oriented ones; go through all ordered pairs of distinct poles and, if the conductivity from the first pole of some pair to the second is different from 0, but the first pole is not connected with the second directly by an elementary oriented circuit, introduce such a circuit, successively applying rule 13; finally, eliminate isolated internal vertices according to rule 1.

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[2] S. V. Yablonskii, *Tr. Matem. inst. im. V. A. Steklova AN SSSR*, **51**, 5 (1958).

Note: Figure translations are in progress. See original paper for figures.