

# ON THE INTEGRATION OF POTENTIAL-IMPULSE FORMS

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**Abstract**

**Full Text**

**CYBERNETICS AND CONTROL THEORY**

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**ON THE INTEGRATION OF POTENTIAL-IMPULSE FORMS**

*(Presented by Academician B. N. Petrov, February 23, 1961)*

The operation of many electronic circuits of discrete action <sup>(1,2)</sup> is based on taking into account changes in the values of logical functions caused by changes in the values of the arguments. To describe circuits of this kind, A. D. Talantsev proposed a potential-impulse form for writing logical conditions and set forth a method for integrating such forms <sup>(3)</sup>.

The present note contains a description of an algebraic method for integrating potential-impulse forms, making it possible to simplify the corresponding circuits.

A potential-impulse form may be represented as a disjunction  $g$  of conjunctions of the form \*

$$\beta_i = x_{i_1}^{p_{i_1}} x_{i_2}^{p_{i_2}} \dots x_{i_{n-1}}^{p_{i_{n-1}}} dx_{i_n}^{p_{i_n}}, \quad (1)$$

i.e.

$$Y = \bigvee_{i=1}^g \beta_i,$$

where  $p_i = 0, 1$ ;  $x_i^{p_i} = x_i p_i \vee \overline{x_i} \overline{p_i}$ ,  $g \leq n \cdot 2^n$ . Each of the conjunctions of form (1) determines which of the variables and how it changes (from 0 to 1 when  $p_i = 0$ , or from 1 to 0 when  $p_i = 1$ ) and what the values of the remaining variables must be in order that the equality  $Y = 1$  hold. We shall call these conjunctions obligatory. It is possible, of course, also to write out such conjunctions for which  $Y = 0$ ; we shall call them forbidden.

However, not all conjunctions not included in (1) are forbidden. There may also be such conjunctions for which the value of  $Y$  is not defined—we shall call them conditional. Taking conditional conjunctions into account in a number of cases makes it possible to simplify an electronic circuit. The integration method set forth proves most effective in the case when there is a large number of conditional conjunctions.

A potential-impulse form is called integrable <sup>(3)</sup> if one can find a Boolean function  $F(x_1, x_2, \dots, x_n)$  such that

$$dF = Y. \quad (2)$$

The function  $F$ , like any Boolean function, may be defined by the set of constituents on which it assumes the value 1. These constituents are called obligatory. Constituents on which the value of the function is equal to 0 are called forbidden. If the function is not defined on certain constituents, then the latter are called conditional.

It is evident that, in order for the function  $F$  to satisfy condition (2), its value must change from one to zero each time when the set of values of the variables  $x_{i_1}^{p_{i_1}}, \dots, x_{i_r}^{p_{i_r}}, \dots, x_{i_n}^{p_{i_n}}$ , determined by the  $i$ -th conjunction of the potential-impulse form  $x_{i_1}^{p_{i_1}} \dots x_{i_{r-1}}^{p_{i_{r-1}}} x_{i_{r+1}}^{p_{i_{r+1}}} \dots$

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\* Here and throughout below we shall assume that the value of no more than one variable changes simultaneously.

$\dots, x_{i_n}^{p_{i_n}} dx_{i_r}^{p_{i_r}}$  passes into a set of other values of these variables  $x_{i_1}^{p_{i_1}}, \dots$

$$\dots, x_{i_r}^{p_{i_r}}, \dots, x_{i_n}^{p_{i_n}},$$

determined by the same conjunction.

The conjunction  $\alpha_i = x_{i_1}^{p_{i_1}} \dots x_{i_r}^{p_{i_r}} \dots x_{i_n}^{p_{i_n}}$  is, therefore, an obligatory constituent of the function  $F$ , while the conjunction  $\alpha'_i = x_{i_1}^{p_{i_1}} \dots x_{i_r}^{\bar{p}_{i_r}} \dots x_{i_n}^{p_{i_n}}$  is forbidden. Let us call these two constituents a pair of constituents and denote it by

$$A_i = \begin{pmatrix} \alpha_i \\ \alpha'_i \end{pmatrix}.$$

Each conjunction of a potential-impulse form, if it is integrable, determines one pair of constituents for the function  $F$ . The potential-impulse form as a whole determines a system of pairs of constituents, which we shall denote by

$$[Y] = [A_{i_1}, \dots, A_{i_l}].$$

It is easy to see that the function  $F$  must have the same value on two sets of values of the variables determined by a forbidden conjunction, since in this case  $Y = 0$ .

We shall say that the constituents

$$\alpha_j = x_{j_1}^{p_{j_1}} \dots x_{j_r}^{p_{j_r}} \dots x_{j_n}^{p_{j_n}},$$

$$\alpha'_j = x_{j_1}^{p_{j_1}} \dots x_{j_r}^{\bar{p}_{j_r}} \dots x_{j_n}^{p_{j_n}}$$

form a link if the values of the function  $F$  on them coincide, i.e. if they are either both obligatory or both forbidden. We shall denote a link of constituents by  $B_j = (\alpha_j \rightarrow \alpha'_j)$ , distinguishing here the first  $\alpha_j$  and the second  $\alpha'_j$  constituents.\*

All forbidden conjunctions determine a system of links of constituents, which we shall represent in the form

$$|Y| = |B_{j_1}|, \dots, |B_{j_s}|.$$

By a dense system of links of constituents we shall mean such a system of links in which the second constituent of each link coincides with the first constituent of another link.

If, in a system of links of constituents, there are links whose first constituents coincide with obligatory constituents of the system of pairs of constituents, then the second constituents of these links must also be assigned to the obligatory ones. The system of pairs of constituents obtained in this way, with the constituents of the system of links included in it as obligatory, will be called the general system of pairs and links of constituents and will be represented in the form

$$\{Y\} = \{A_{i_1}, \dots, A_{i_n}, B_{j_1}, \dots, B_{j_h}\}, \quad h \leq s.$$

If, in the general system of pairs and links of constituents, none of the obligatory constituents belongs to the number of forbidden ones, then we shall call such a system consistent. If it turns out that the general system of pairs and links of constituents is not consistent, but can be divided into  $m > 1$  consistent subsystems in such a way that, in this division, a pair of constituents and the obligatory constituents assigned to it from the system of links are in one subsystem, then such a general system is partially consistent. Each of the resulting subsystems will correspond to its own function  $F^j$ .

We shall call the potential-impulse form (1) partially integrable if one can find a set of functions  $F^j$  such that

$$\bigvee_{j=1}^m dF^j = Y. \quad (3)$$

\* It should be noted that if the second constituent is obligatory and the first forbidden, then the conjunction defining this link will also be forbidden.

If it is impossible to find a single function satisfying (2), or a set of functions  $F^j$  satisfying (3), then we shall call the potential-impulse form nonintegrable.

There are cases when an inconsistent general system of pairs and links of constituents cannot be divided into several consistent subsystems; such a general system will be called absolutely inconsistent. This can occur when to some pair of constituents there must be assigned such a dense system of links in which one of the second constituents coincides with the forbidden constituents of the given pair.

Using the definitions introduced, one can formulate the following theorems.

**Theorem 1.** *A potential-impulse form is integrable if there corresponds to it a consistent general system of pairs and links of constituents.*

**Theorem 2.** *A potential-impulse form is partially integrable if there corresponds to it a partially consistent general system of pairs and links of constituents.*

**Theorem 3.** *A potential-impulse form is nonintegrable if there corresponds to it an absolutely inconsistent general system of pairs and links of constituents.*

Let us consider several examples of integration of potential-impulse forms. In doing so we shall assume that, if no forbidden conjunctions are specified, then all the remaining conjunctions, except those determined by the potential-impulse form, are conditional. In this case we do not have a system of links of constituents, and the general system of pairs and links of constituents will coincide with the system of pairs of constituents.

**Example 1.** A potential-impulse form is given:

$$Y = \bar{x}_2\bar{x}_3 d\bar{x}_1 \vee x_2x_3 d\bar{x}_1 \vee x_1\bar{x}_3 d\bar{x}_2 \vee \bar{x}_2x_3 dx_1 \vee \bar{x}_1x_2 d\bar{x}_3 \vee x_1x_3 dx_2 \vee \bar{x}_1x_3 d\bar{x}_2.$$

We compose the system of pairs of constituents

$$[Y] = \{Y\} = \left\{ \begin{array}{l} (x_2\bar{x}_3\bar{x}_1); (x_2x_3\bar{x}_1); (x_1\bar{x}_3\bar{x}_2); \\ (\bar{x}_2\bar{x}_3x_1); (x_2x_3x_1); (x_1\bar{x}_3x_2); \\ (\bar{x}_2x_3x_1); (\bar{x}_1x_2\bar{x}_3); (x_1x_3\bar{x}_2); (\bar{x}_1x_3\bar{x}_2) \end{array} \right\}.$$

This system is not consistent, since, for example, the required constituent of the third pair coincides with the forbidden constituent of the first pair, the required constituent of the second pair coincides with the forbidden constituent of the fifth and seventh pairs, etc. However, it is easy to see that this inconsistent system can be divided into two consistent ones; consequently, the given potential-impulse form is partially integrable. For example, the first, second, and fourth

pairs of constituents may be assigned to one subsystem, and all the remaining pairs to another. We obtain the following two consistent subsystems:

$$\{Y\}_1 = \left\{ \begin{pmatrix} \bar{x}_1 \bar{x}_2 \bar{x}_3 \\ x_1 \bar{x}_2 \bar{x}_3 \end{pmatrix}; \begin{pmatrix} \bar{x}_1 x_2 x_3 \\ x_1 x_2 x_3 \end{pmatrix}; \begin{pmatrix} x_1 \bar{x}_2 x_3 \\ x_1 x_2 x_3 \end{pmatrix} \right\},$$

$$\{Y\}_2 = \left\{ \begin{pmatrix} x_1 \bar{x}_2 \bar{x}_3 \\ x_1 x_2 \bar{x}_3 \end{pmatrix}; \begin{pmatrix} \bar{x}_1 x_2 \bar{x}_3 \\ \bar{x}_1 x_2 x_3 \end{pmatrix}; \begin{pmatrix} x_1 x_2 x_3 \\ x_1 \bar{x}_2 x_3 \end{pmatrix}; \begin{pmatrix} \bar{x}_1 \bar{x}_2 x_3 \\ \bar{x}_1 x_2 x_3 \end{pmatrix} \right\}.$$

To the subsystem  $\{Y\}_1$  there corresponds the function

$$F^1 = \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 x_2 x_3 \vee x_1 \bar{x}_2 x_3.$$

To the subsystem  $\{Y\}_2$  there corresponds

$$F^2 = x_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 x_2 \bar{x}_3 \vee x_1 x_2 x_3 \vee \bar{x}_1 \bar{x}_2 x_3,$$

whence

$$Y = dF^1 \vee dF^2 = d(\bar{x}_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 x_2 x_3 \vee x_1 \bar{x}_2 x_3) \vee \\ \vee d(x_1 \bar{x}_2 \bar{x}_3 \vee \bar{x}_1 \bar{x}_2 \bar{x}_3 \vee x_1 x_2 x_3 \vee \bar{x}_1 \bar{x}_2 x_3).$$

### Example 2.

$$Y = x_1 \bar{x}_2 dx_3 \vee \bar{x}_1 x_3 dx_2 \vee x_1 x_2 d\bar{x}_3.$$

In addition to the potential-impulse form, the forbidden conjunctions are given:  $\bar{x}_2 x_3 dx_1$ ,  $\bar{x}_1 \bar{x}_2 dx_3$ ,  $\bar{x}_2 \bar{x}_3 d\bar{x}_1$ . We compose the system of pairs of constituents

$$[Y] = \left[ \begin{pmatrix} x_1 \bar{x}_2 x_3 \\ x_1 x_2 x_3 \end{pmatrix}; \begin{pmatrix} \bar{x}_1 x_3 x_2 \\ \bar{x}_1 \bar{x}_2 x_3 \end{pmatrix}; \begin{pmatrix} x_1 x_2 \bar{x}_3 \\ x_1 x_2 x_3 \end{pmatrix} \right]$$

and the system of connections of the constituents

$$|Y| = (x_1 \bar{x}_2 x_3 \rightarrow \bar{x}_1 \bar{x}_2 x_3); (\bar{x}_1 \bar{x}_2 x_3 \rightarrow \bar{x}_1 \bar{x}_2 \bar{x}_3); (\bar{x}_1 x_2 \bar{x}_3 \rightarrow x_1 \bar{x}_2 \bar{x}_3).$$

From  $[Y]$  and  $|Y|$  we obtain:

$$\{Y\} = \left\{ \begin{pmatrix} x_1 \bar{x}_2 x_3 \\ x_1 x_2 x_3 \end{pmatrix}; \begin{pmatrix} \bar{x}_1 x_2 x_3 \\ \bar{x}_1 \bar{x}_2 x_3 \end{pmatrix}; \begin{pmatrix} x_1 x_2 \bar{x}_3 \\ x_1 x_2 x_3 \end{pmatrix}; (x_1 \bar{x}_2 x_3 \rightarrow \bar{x}_2 \bar{x}_3 \bar{x}_1) \right\},$$

$$(\bar{x}_1\bar{x}_2x_3 \rightarrow \bar{x}_1\bar{x}_2\bar{x}_3); (\bar{x}_1x_2\bar{x}_3 \rightarrow x_1\bar{x}_2\bar{x}_3)\}.$$

The resulting general system  $\{Y\}$  is inconsistent, but it cannot be divided into several consistent ones, since the system of connections assigned to the first pair of constituents is dense, and the second constituent of the third connection coincides with the forbidden constituent of the pair of constituents under consideration. Therefore the system  $\{Y\}$  is absolutely inconsistent, and the potential-impulse form  $Y$  is not integrable.

In conclusion, the authors consider it their pleasant duty to express their gratitude to M. L. Tsetlin for his interest in the work and for valuable comments.

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## CITED LITERATURE

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<sup>2</sup> M. L. Tsetlin, DAN, 117, No. 6, 979 (1957).

<sup>3</sup> A. D. Talantsev, *Avtomatika i telemekh.*, 20, No. 7, 898 (1959).

*Note: Figure translations are in progress. See original paper for figures.*

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